Question 1

(a) We may write equations for each of the key nodes C, D, A, B. For E and F, we can treat these as simple voltage dividers.

Node C:
$$4 - 2 + \frac{V_C}{5+15} + \frac{V_C - V_A}{5} = 0$$
$$40 + V_C + 4V_C - 4V_A = 0$$
$$-4V_A + 5V_C = -40 \qquad (1)$$

Node D:
$$2 - 5 + \frac{V_D}{15+5} + \frac{V_D - V_B}{5} = 0$$
$$-60 + V_D + 4V_D - 4V_B = 0$$
$$-4V_B + 5V_D = 60 \qquad (2)$$

Nodes A and B form a supernode.

$$\frac{V_A - V_C}{5} + \frac{V_B - V_D}{5} = 0$$
$$V_A + V_B - V_C - V_D = 0 \qquad (3)$$

and $V_A - V_B = 5$

or $V_A = V_B + 5 \qquad (4)$

From here, there are many ways to proceed. Let's add equations (1) and (2) with (3):

$$-4V_A - 4V_B + 5V_C + 5V_D = 20 \qquad (1)+(2)$$
$$4V_A + 4V_B - 4V_C - 4V_D = 0 \qquad (3)\times 4$$

$$V_C + V_D = 20 \qquad (5)$$

Therefore, (3) becomes $V_A + V_B = 20 \qquad (6)$

Using (4), equation (6) becomes

$$V_B + 5 + V_B = 20$$
$$2V_B = 15$$

so $\boxed{V_B = 7.5}$

and $V_A = V_B + 5$, so $\boxed{V_A = 12.5}$

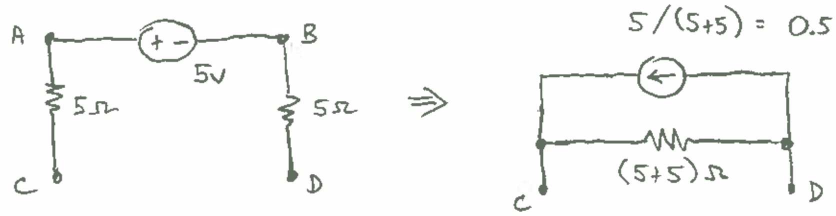
Then, from (1) $V_C = \dfrac{-40 + 4V_A}{5} = \dfrac{-40 + 50}{5}$

$$\boxed{V_C = 2}$$

and from (5), $V_D = 20 - V_C$, so $\boxed{V_D = 18}$

An alternative solution could be to recognize that the top portion of the circuit can be redrawn using a source transformation

$5/(5+5) = 0.5\text{ A}$



The equations for C and D become

Node C: $4 - 2 + \dfrac{V_C}{5+15} - 0.5 + \dfrac{V_C - V_D}{10} = 0$

$$40 + V_C - 10 + 2V_C - 2V_D = 0$$
$$3V_C - 2V_D = -30 \qquad (7)$$

Node D: $2 - 5 + \dfrac{V_D}{15+5} + 0.5 + \dfrac{V_D - V_C}{10} = 0$

$$-60 + V_D + 10 + 2V_D - 2V_C = 0$$
$$3V_D - 2V_C = 50 \qquad (8)$$

Solving (7) and (8)

$$6V_C - 4V_D = -60 \qquad (7)\times 2$$
$$-6V_C + 9V_D = 150 \qquad (8)\times 3$$

$5V_D = 90$, so $V_D = 18$

and $V_C = 2$.

V_A and V_B can then be found using (3) and (4)

Finally, using the voltage-divider principle,

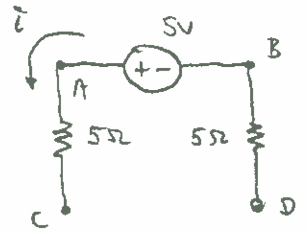
$$V_E = \frac{15}{5+15} \times V_C = 1.5\ \text{V}$$

$$V_F = \frac{5}{5+15} \times V_D = 4.5\ \text{V}$$

(b) Power in all circuit elements

<u>Sources</u>:

$P_{4A} = 4A \times V_C = +8\ \text{W}$ (absorb)
$P_{5A} = -5A \times V_D = -90\ \text{W}$ (supply)
$P_{2A} = -2A \times (V_C - V_D) = +32\ \text{W}$ (absorb)



$$i = \frac{V_A - V_C}{5} = \frac{10.5}{5} = 2.1\ \text{A}$$

$$P_{5V} = -i \times 5 = -10.5\ \text{W}\ \text{(supply)}$$

<u>Resistors</u>:

Connected to node E:

$$P_{CE} = \frac{(V_C - V_E)^2}{5} = 0.05\ \text{W}$$

$$P_{EO} = \frac{V_E^2}{15} = 0.15\ \text{W}$$

Connected to node F:

$$P_{DF} = \frac{(V_D - V_F)^2}{15} = 12.15\ \text{W}$$

$$P_{FO} = \frac{V_F^2}{5} = 4.05\ \text{W}$$

Top portion of circuit:

$$P_{AC} = \frac{(V_A - V_C)^2}{5} = 22.05\ \text{W}$$

$$P_{BD} = \frac{(V_B - V_D)^2}{5} = 22.05\ \text{W}$$

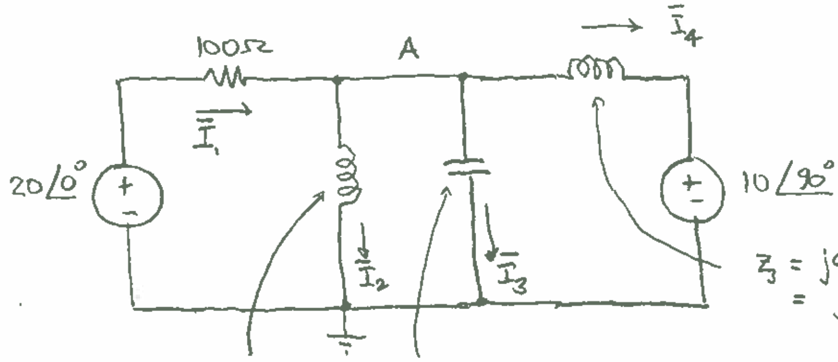
Energy balance:

$$\underbrace{8 - 90 + 32 - 10.5}_{\text{sources} = -60.5\text{W}} + \underbrace{0.05 + 0.15 + 12.15 + 4.05 + 2 \times 22.05}_{\text{resistors} = +60.5\text{W}}$$

$$= 0 \quad \text{(good!)}$$

Question 2

Redrawing the circuit with phasors and complex impedances



$Z_3 = j\omega \times 0.1 = j100\,\Omega$

$Z_1 = j\omega \times 0.005 = j5\,\Omega$

$Z_2 = \dfrac{1}{j\omega \times 200 \times 10^{-6}} = -j5\,\Omega$

(a) One node-voltage equation at node A

$$\frac{\bar{V}_A - 20}{100} + \cancel{\frac{\bar{V}_A}{j5}} + \cancel{\frac{\bar{V}_A}{-j5}} + \frac{\bar{V}_A - 10\angle 90^\circ}{j100} = 0$$

$$\bar{V}_A - 20 + \frac{\bar{V}_A - j10}{j} = 0$$

$$j(\bar{V}_A - 20) + \bar{V}_A - j10 = 0$$

$$\bar{V}_A(1 + j) = j30$$

Thus, $\bar{V}_A = \dfrac{j30}{1+j} = \dfrac{j30}{1+j} \cdot \dfrac{1-j}{1-j}$

$$= \frac{j30 + 30}{2}$$

$$\bar{V}_A = 15 + j15$$

Phasor currents:

$$\bar{I}_1 = \frac{20 - \bar{V}_A}{100} = 0.05 - j0.15 = 0.158\angle -71.57^\circ$$

$$\bar{I}_2 = \frac{\bar{V}_A}{j5} = \frac{15 + j15}{j5} = \frac{3 + j3}{j} \times \frac{-j}{-j} = 3 - j3$$

$$= 4.24\angle -45^\circ$$

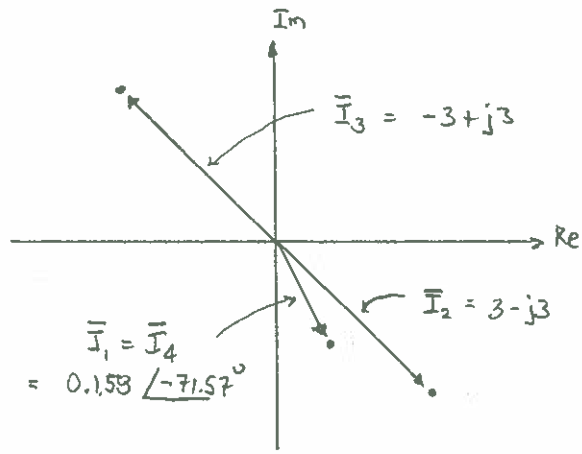
$$\bar{I}_3 = \frac{\bar{V}_A}{-j5} = -\bar{I}_2 = -3 + j3 = 4.24 \angle 135^\circ$$

$$\bar{I}_4 = \frac{\bar{V}_A - j10}{j100} = \frac{15 + j15 - j10}{j100} = \frac{15 + j5}{j100}$$

$$= 0.05 - j0.15 = 0.158 \angle 71.57^\circ$$

Quick check: Since $\bar{I}_2$ and $\bar{I}_3$ cancel at node A, then $\bar{I}_4$ must be the same as $\bar{I}_1$. (good!)

Phasor diagram:



(b) We have $\theta_V = 0^\circ$ and $\theta_I = -71.57^\circ$, so power angle $\theta = \theta_V - \theta_I = 71.57^\circ$

Apparent power $= V_{1(rms)} \times I_{1(rms)}$

$$= \frac{20}{\sqrt{2}} \times \frac{0.158}{\sqrt{2}} = 1.58 \text{ VA}$$

Active (average) power $P = 1.58 \cos\theta$
$= 0.5$ W

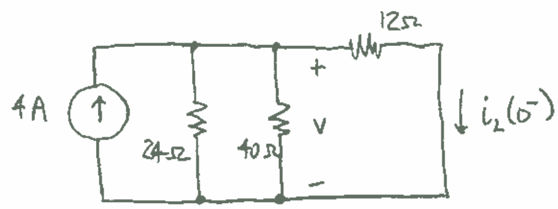
Reactive power $Q = 1.58 \sin\theta$
$= 1.5$ VAR

Power factor $\cos\theta = 0.316$

Since $\theta_I < \theta_V$, the power factor is 0.316 lagging.

Question 3

(a) In steady-state at $t = 0^-$, the equivalent circuit is



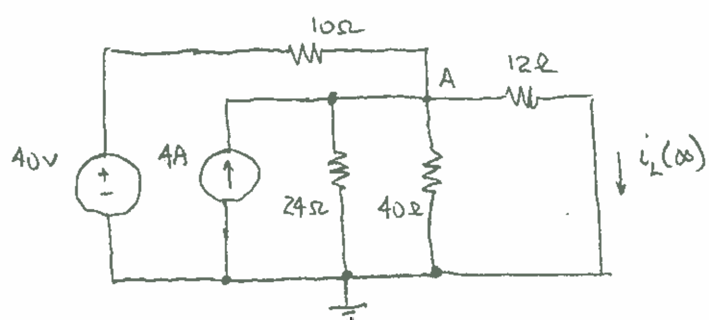
Three parallel resistors: $R_{eq} = \left[\frac{1}{24} + \frac{1}{40} + \frac{1}{12}\right]^{-1}$

$= 6.667\ \Omega$

Voltage $v = 4R_{eq} = 26.667\ \text{v}$

giving $i_L(0^-) = \dfrac{v}{12} = 2.222\ \text{A}$

In steady-state when $t \to \infty$, the equivalent circuit is



At node A: $-4 + \dfrac{V_A}{24} + \dfrac{V_A}{40} + \underbrace{\dfrac{V_A}{12}}_{\text{this is } i_L(\infty)} + \dfrac{V_A - 40}{10} = 0$

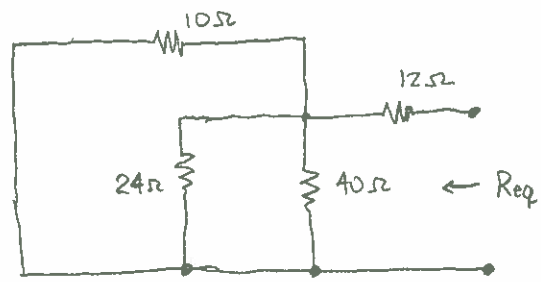
$(\times 120)$ $\quad -480 + 5V_A + 3V_A + 10V_A + 12V_A - 480 = 0$

$30\ V_A = 960$

$V_A = 32\ \text{V}$

Therefore, $i_L(\infty) = \dfrac{V_A}{12} = 2.667\ \text{A}$

We may zero the sources to determine the total resistance across the inductor, $t > 0$.



$$R_{eq} = 10 // 24 // 40 + 12$$
$$= 6 + 12$$
$$= 18\,\Omega$$

Time constant $\tau = L/R = \dfrac{0.09}{18} = 0.005\text{ s}$ (5 ms)

The charging curve for $t \geq 0$ is given by

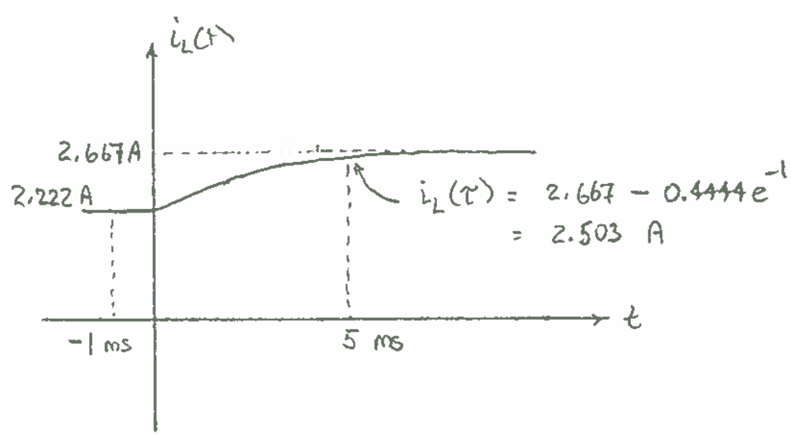
$$i_L(t) = K_1 + K_2 e^{-t/\tau}$$

At $t = 0^-$, $i_L(0^-) = 2.222 = K_1 + K_2$
At $t \to \infty$, $i_L(\infty) = 2.667 = K_1$

so $K_1 = 2.667$, $K_2 = -0.4444$

Finally,
$$\boxed{i_L(t) = 2.667 - 0.4444e^{-t/0.005}}$$

Sketch:



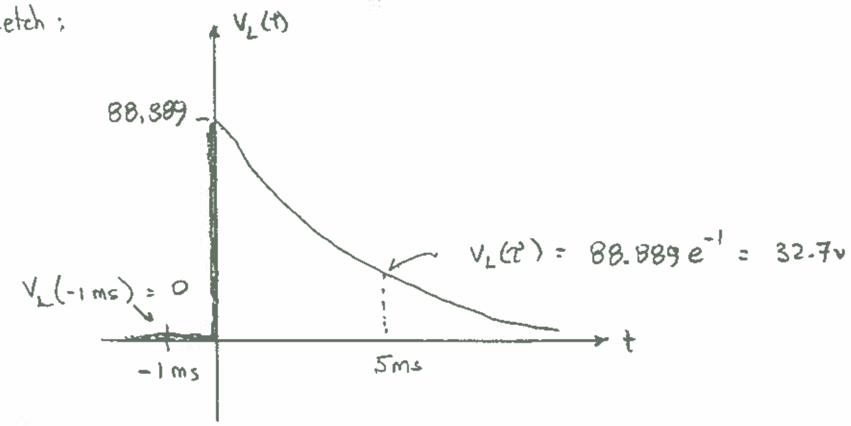
(b) For the inductor voltage, we may simply use

$$V_L(t) = L\frac{di_L(t)}{dt} = 0.09\frac{d}{dt}\left[2.667 - 0.4444e^{-t/\tau}\right]$$

$$= 0.09 \times \frac{0.4444}{\tau}e^{-t/\tau}$$

$$V_L(t) = 88.889\, e^{-t/0.005} \text{ V}, \quad t \geq 0$$

Sketch:



(c) When the switch is re-opened at 6 ms, we have

$$i_L(6\text{ ms}) = 2.667 - 0.4444\, e^{-6/5}$$
$$= 2.538 \text{ A}$$

This will be the initial current at $t = 6$ ms, which then decays as $t \to \infty$ to the value we determined for $i_L(0^-)$.

At $t = 6$ms, $\quad i_L(t) = 2.538 = K_1 + K_2 e^{-(t-0.006)/0.005}$
$$= K_1 + K_2$$

At $t \to \infty \quad i_L(\infty) = 2.222 = K_1$

so $K_1 = 2.222$
$K_2 = 0.316$

And $\quad i_L(t) = 2.222 + 0.316\, e^{-(t-0.006)/0.005}$

$$= 2.222 + 0.316 e^{-6/5} e^{-t/\tau}$$

$$= 2.229 + 0.0952\, e^{-t/\tau}, \quad t \geq 6 \text{ ms}$$

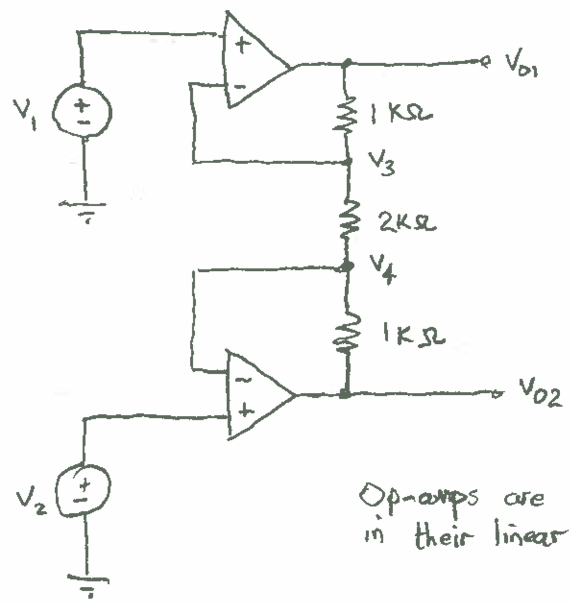
Time constant will also be different, $t \geq 6$ ms

$$R_{eq} = 24 /\!/ 40 + 12$$
$$= 15 + 12 = 27\,\Omega$$

so $\tau = L/R = 0.09/27$
$$= 3.333 \text{ ms}$$

Question 4

(a)



Op-amps are operating in their linear regions

The summing-point constraints impose

$$V_3 = V_1 \quad \text{and} \quad V_4 = V_2$$

Using the node-voltage method,

Node V_3 : $$\frac{V_3 - V_{o1}}{1000} + \frac{V_3 - V_4}{2000} = 0$$

so $$2(V_1 - V_{o1}) + V_1 - V_2 = 0$$

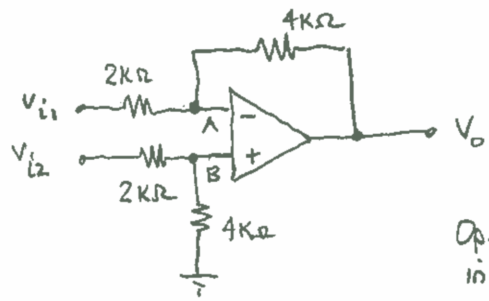
and $$V_{o1} = \frac{3V_1 - V_2}{2}$$

Node V_4 : $$\frac{V_4 - V_{o2}}{1000} + \frac{V_4 - V_3}{2000} = 0$$

so $$2(V_2 - V_{o2}) + V_2 - V_1 = 0$$

and $$V_{o2} = \frac{3V_2 - V_1}{2}$$

(b)



Op-amp operating in linear region

Node A: $$\frac{V_A - V_{i1}}{2000} + \frac{V_A - V_o}{4000} = 0$$

$$2V_A - 2V_{i1} + V_A - V_o = 0 \qquad (1)$$

Node B: $$\frac{V_B - V_{i2}}{2000} + \frac{V_B}{4000} = 0$$

$$2V_B - 2V_{i2} + V_B = 0$$
$$3V_B = 2V_{i2}$$

$$\text{so} \quad V_B = \frac{2}{3} V_{i2} \qquad (2)$$

The summing-point constraints impose $V_A = V_B$, so

$$V_A = \frac{2}{3} V_{i2}$$

and from (1),

$$3\left(\frac{2}{3} V_{i2}\right) - 2V_{i1} = V_o$$

$$V_o = 2\left(V_{i2} - V_{i1}\right)$$

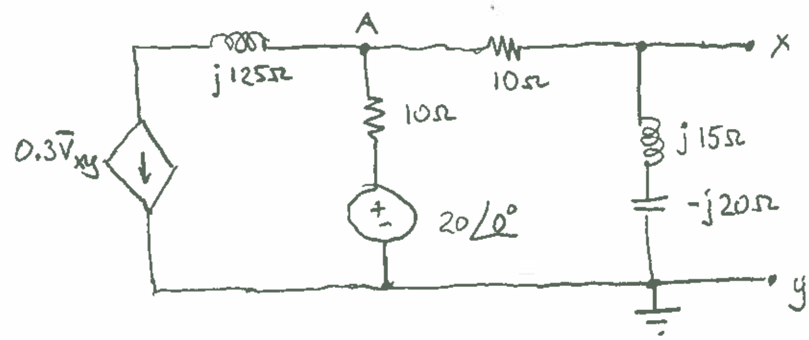
Using $V_{i1} = V_{o1} = \dfrac{3V_1 - V_2}{2}$ and $V_{i2} = V_{o2} = \dfrac{3V_2 - V_1}{2}$,

$$V_o = 2\left[\frac{3V_2 - V_1}{2} - \frac{3V_1 - V_2}{2}\right]$$

Finally, $$\boxed{V_o = 4\left(V_2 - V_1\right)}$$

Question 5

(a) First find $V_T = V_{oc}$.



Node-voltage method:

Node A: $$0.3\bar{V}_{xy} + \frac{\bar{V}_A - 20}{10} + \frac{\bar{V}_A - \bar{V}_x}{10} = 0$$

[Note: $\bar{V}_{xy} = \bar{V}_x$]

$$3\bar{V}_x + \bar{V}_A - 20 + \bar{V}_A - \bar{V}_x = 0$$

$$2\bar{V}_A + 2\bar{V}_x = 20 \qquad (1)$$

Node x: $$\frac{\bar{V}_x - \bar{V}_A}{10} + \frac{\bar{V}_x}{j15 - j20} = 0$$

$$\bar{V}_x - \bar{V}_A + \frac{2\bar{V}_x}{-j} = 0$$

$$\bar{V}_x(1 + 2j) = \bar{V}_A \qquad (2)$$

Substitute $\bar{V}_A$ from (2) into (1)

$$2\bar{V}_x(1+2j) + 2\bar{V}_x = 20$$

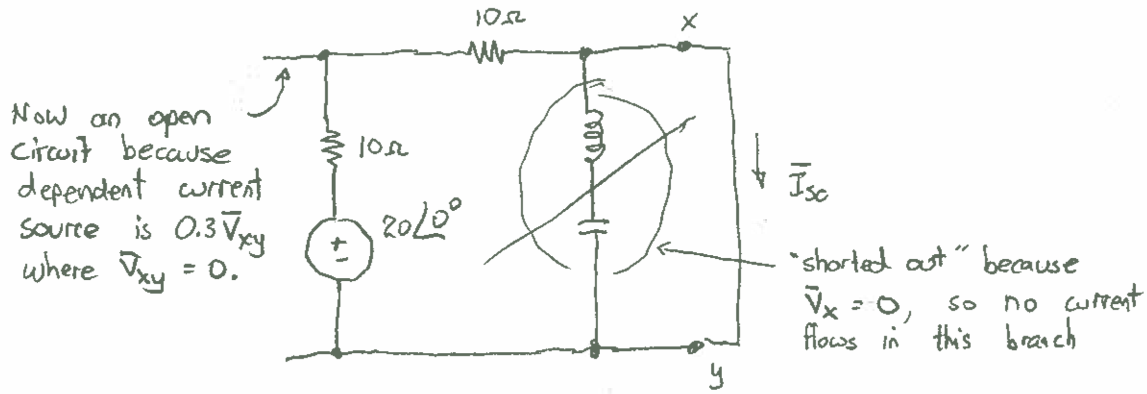
$$(4 + j4)\bar{V}_x = 20$$

$$\bar{V}_x = \frac{20}{4+j4} = \frac{5}{1+j} \cdot \frac{1-j}{1-j}$$

$$= \frac{5 - j5}{2} = 2.5 - j2.5$$

Thus, $\bar{V}_T = \bar{V}_{oc} = \bar{V}_x = 2.5 - j2.5$

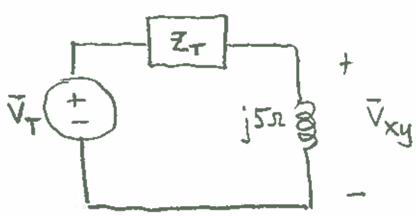
Since there is a dependent source, we must calculate $\bar{I}_{sc}$. With x and y connected together, the circuit simplifies to:



By inspection, $\bar{I}_{sc} = \dfrac{20\angle 0^\circ}{10+10} = 1$

Therefore, $Z_T = \dfrac{\bar{V}_T}{\bar{I}_{sc}} = 2.5 - j2.5$

(b) Using the newly found Thevenin equivalent

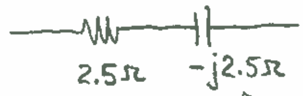


Voltage divider:
$$\bar{V}_{xy} = \frac{j5}{Z_T + j5} \times (2.5 - j2.5)$$
$$= \frac{j5}{2.5 - j2.5 + j5} \times (2.5 - j2.5)$$
$$= \frac{j5}{2.5 + j2.5} \times (2.5 - j2.5)$$
$$= \frac{5\angle 90^\circ}{2.5\sqrt{2}\angle 45^\circ} \times 2.5\sqrt{2}\angle -45^\circ$$
$$= 5\angle 0^\circ$$

In the time domain, $v_{xy}(t) = 5\cos(2000t)$

(c) We have $Z_T = 2.5 - j2.5\ \Omega$

negative, so capacitive reactance



$$\frac{1}{j\omega C} = -j2.5$$

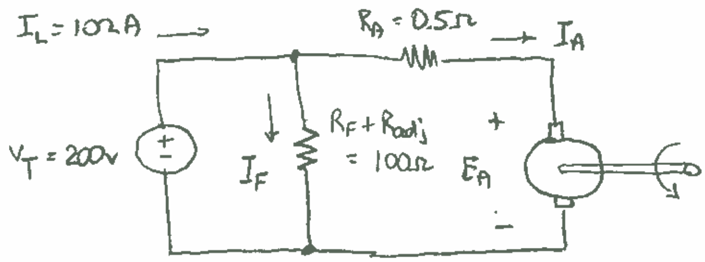
$$\text{so} \quad C = \frac{1}{2.5\,\omega} = \frac{1}{2.5 \times 2000}$$

$$= 0.0002\ \text{F}$$

$$(200\ \mu\text{F})$$

Question 6

The motor's equivalent circuit:



$T_{out} = 40$ Nm

$T_{rot} = 2$ Nm

(a) $I_F = \dfrac{200}{R_F + R_{adj}} = \dfrac{200}{100} = 2$ A

$I_A = I_L - I_F = 100$ A

(b) Total developed torque $\quad T_{dev} = T_{out} + T_{rot}$

$= 42$ Nm

The machine constant can be determined

$$T_{dev} = K\phi I_A, \quad \text{so} \quad K\phi = \frac{T_{dev}}{I_A} = \frac{42}{100}$$

$$= 0.42$$

The speed may be determined using the other machine equation. First,

$$E_A = V_T - I_A R_A$$

$$= 200 - 100\,(0.5) = 150\ \text{V}$$

$$E_A = K\phi\,\omega_m \quad , \quad \text{so} \quad \omega_m = \frac{E_A}{K\phi}$$

$$\omega_m = \frac{150}{0.42} = 357.14 \text{ r/s}$$

$$\text{so} \quad n_m = \omega_m \times \frac{60}{2\pi} = 3410.5 \text{ rpm}$$

(c) Developed power $P_{dev} = T_{dev} \cdot \omega_m$

$$= 15{,}000 \text{ W}$$
$$= 20.1 \text{ HP}$$

Output power $P_{out} = T_{out} \cdot \omega_m$

$$= 14{,}286 \text{ W}$$
$$= 19.15 \text{ HP}$$

(d) Power losses

$$P_{field\text{-}loss} = I_F^2 (R_F + R_{adj})$$
$$= 400 \text{ W}$$

$$P_{arm\text{-}loss} = I_A^2 R_A$$
$$= 5000 \text{ W}$$

(e) Efficiency

$$P_{in} = V_T \cdot I_L = 20{,}400 \text{ W}$$

$$P_{out} = 14{,}286 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 70\%$$