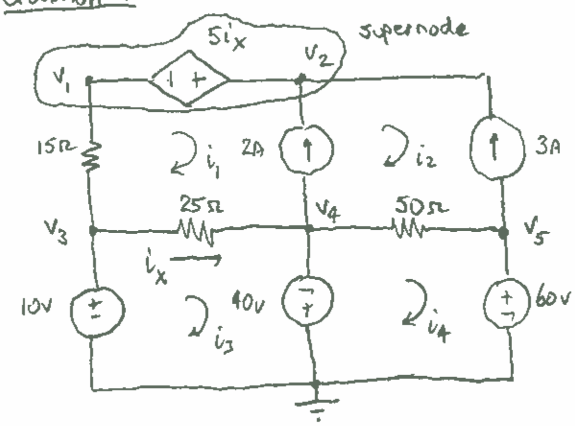
Question 1



(a) By inspection,

$$\boxed{\begin{aligned} V_3 &= 10 \\ V_4 &= -40v \\ V_5 &= 60v \end{aligned}}$$

V_1 and V_2 form a supernode

Supernode equation: $\frac{V_1 - V_3}{15} - 2 - 3 = 0$

so $V_1 - V_3 = 75$, and $V_1 = 75 + V_3 = \boxed{85v}$

Supernode dependence: $V_2 - V_1 = 5i_x$

where $i_x = \frac{V_3 - V_4}{25}$

Combining: $V_2 - V_1 = 5\left(\frac{V_3 - V_4}{25}\right) = 5\left(\frac{10+40}{25}\right)$

so $V_2 - V_1 = 10$ (2)

Substituting $V_1 = 85v$, $V_2 = 10 + V_1$
$\boxed{= 95\ v.}$

(b) By inspection, $\boxed{i_2 = -3A.}$

We may write mesh equations for the other meshes or use our node-voltage calculations.

We have $i_1 = \frac{V_3 - V_1}{15} = \frac{10 - 85}{15} = \boxed{-5A}$

In mesh 3: $-10 + 25(i_3 - i_1) - 40 = 0$
$25(i_3 + 5) = 50$
$25 i_3 = -75$

$$\boxed{i_3 = -3\ A}$$

In mesh 4: $40 + 50(i_4 - i_2) + 60 = 0$

$$50(i_4 + 3) = -100$$
$$50 i_4 = -250$$
$$\boxed{i_4 = -5 \text{ A}}$$

(c) $P_{15\Omega} = i_1^2(15) = (-5)^2(15) = 375 \text{ W}$

$P_{25\Omega} = \dfrac{(V_3 - V_4)^2}{25} = \dfrac{(10 + 40)^2}{25} = 100 \text{ W}$

$P_{50\Omega} = \dfrac{(V_4 - V_5)^2}{50} = \dfrac{(-40 - 60)^2}{50} = 200 \text{ W}$

resistances - power always absorbed.

$P_{10V} = -i_3 \times 10 = 3 \times 10 = 30 \text{ W}$ (absorbed)

$P_{40V} = (i_4 - i_3) \times 40 = (-5 + 3) \times 40 = -80 \text{ W}$ (delivered)

$P_{60V} = i_4 \times 60 = -5 \times 60 = -300 \text{ W}$ (delivered)

$P_{2A} = (V_4 - V_2) \times 2 = (-40 - 95) \times 2 = -270 \text{ W}$ (delivered)

$P_{3A} = (V_5 - V_2) \times 3 = (60 - 95) \times 3 = -105 \text{ W}$ (delivered)

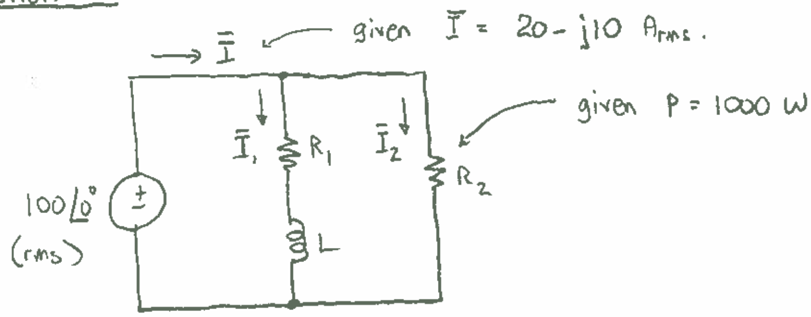
$P_{5i_x} = -i_1 \times 5 i_x = -5 i_1 \left(\dfrac{V_3 - V_4}{25}\right) = -10 i_1 = 50 \text{ W}$ (absorbed)

Check: $\underbrace{375 + 100 + 200 + 30 + 50}_{\text{absorbed}} \underbrace{- 80 - 300 - 270 - 105}_{\text{delivered}}$

$= 0 \text{ W}$ (energy balance)

Question 2

(a)



We can start with R_2. We know $P = \frac{V_{rms}^2}{R_2}$, so $R_2 = \frac{V_{rms}^2}{P}$

$$R_2 = \frac{(100)^2}{1000} \leftarrow \text{already rms}$$

$$\boxed{R_2 = 10\,\Omega}$$

This tells us that $\bar{I}_2 = \frac{100\angle 0°}{10\,\Omega} = 10\angle 0°$ A (rms)

KCL at the top node gives $\bar{I} = \bar{I}_1 + \bar{I}_2$, so $\bar{I}_1 = \bar{I} - \bar{I}_2$

$$\bar{I}_1 = 20 - j10 - 10 = 10 - j10 \text{ A (rms)}$$

In the middle branch, we have

$$\bar{I}_1 = \frac{100\angle 0°}{Z_{eq}} = \frac{100}{R_1 + j\omega L} = 10 - j10 \text{ A (rms)}$$

Therefore, $Z_{eq} = \frac{100}{\bar{I}_1} = \frac{100}{10 - j10}$

$$= \frac{100\,(10 + j10)}{(10 - j10)(10 + j10)}$$

$$= \frac{1000 + j1000}{100 + 100}$$

so $Z_{eq} = 5 + j5 = R_1 + j\omega L$.

From this, $\boxed{R_1 = 5\,\Omega}$ and $\omega L = 5\,\Omega$

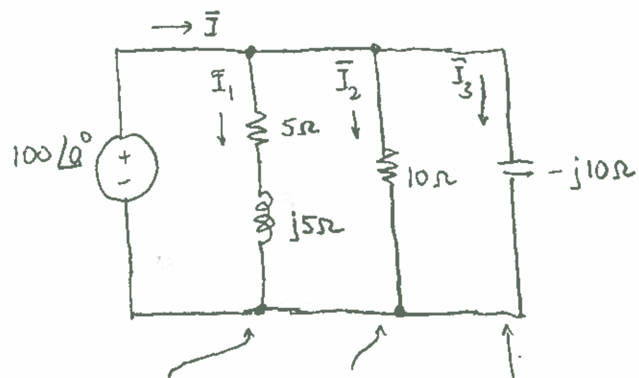
With a frequency of 60 Hz, $2\pi(60)L = 5$

$$\boxed{L = 13.27 \text{ mH}}$$

(b) Connecting the capacitor, where $Z_c = \dfrac{1}{j\omega C} = \dfrac{1}{j(2\pi)(60)(265.26 \times 10^{-6})}$

so $Z_c = -j10\Omega$

Circuit:

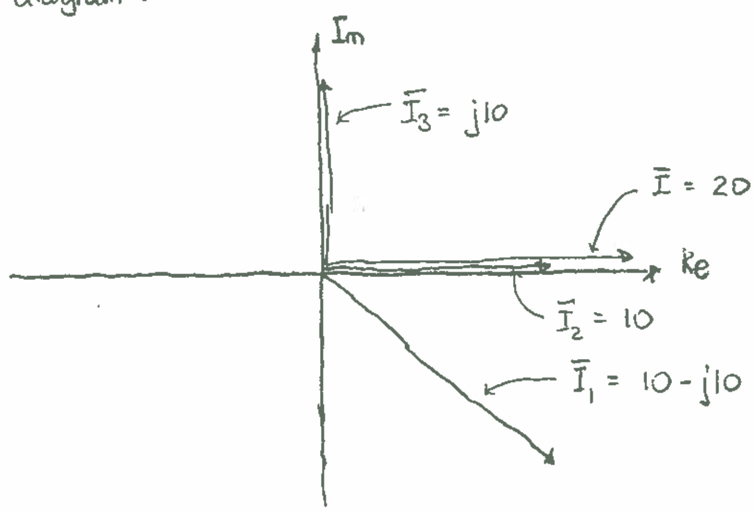


$\bar{I}_1 = 10 - j10$ $\bar{I}_2 = 10$ $\bar{I}_3 = \dfrac{100}{-j10} = j10$

Total current: $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

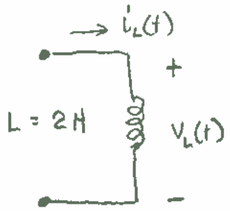
$= 10 - j10 + 10 + j10 = 20\angle 0^\circ$ A (rms)

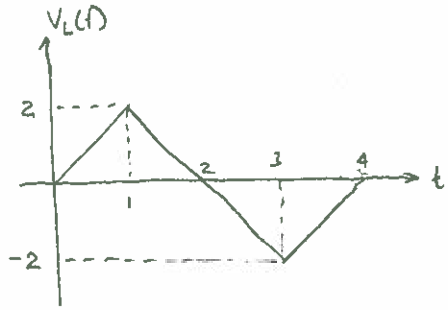
Phasor diagram:



Question 3

We have





(a) For the inductor, the voltage-current relationship is defined as

$$v_L(t) = L\frac{di_L(t)}{dt} \quad \text{and} \quad i_L(t) = \frac{1}{L}\int_{t_1}^{t_2} v_L(\tau)\,d\tau + V_L(t_1)$$

Dividing the voltage waveform into 3 segments ...

<u>$0 \le t \le 1$</u> : From the above graph, $v_L(t) = \underbrace{2t}_{\text{slope} = 2}$

Then, $i_L(t) = \frac{1}{L}\int_0^t (2\tau)\,d\tau + \underbrace{i_L(0)}_{=0}$

$$= \frac{1}{2}\left[\frac{1}{2}\cdot 2\tau^2\right]_0^t + 0$$

$$= \frac{1}{2}(t^2 - 0) = \boxed{\frac{1}{2}t^2}$$

<u>$1 \le t \le 3$</u> : $v_L(t) = -2t + b$

slope = −2 ; intercept: at $t = 2$, $v_L(2) = 0 = -2\times 2 + b$
so $b = 4$.

$$v_L(t) = -2t + 4$$

Then, $i_L(t) = \frac{1}{2}\int_1^t (-2\tau + 4)\,d\tau + i_L(1)$

from previous interval, $i_L(1) = \frac{1}{2}\times 1^2 = \frac{1}{2}$ A

$$i_L(t) = \frac{1}{2}\left[-\tau^2 + 4\tau\right]_1^t + \frac{1}{2}$$

$$= \frac{1}{2}\left(-t^2 + 4t + 1 - 4\right) + \frac{1}{2}$$

$$\boxed{= -\frac{1}{2}t^2 + 2t - 1}$$

$\underline{3 \le t \le 4}$:

$$v_L(t) = 2t + b$$

slope = 2 intercept: at $t = 4$, $v_L(4) = 0 = 2 \times 4 + b$
so $b = -8$

$$v_L(t) = 2t - 8$$

Then,

$$i_L(t) = \frac{1}{2}\int_3^t (2\tau - 8)\,d\tau + i_L(3)$$

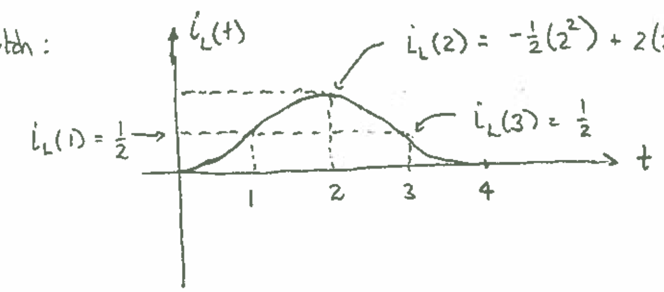
from previous interval,
$i_L(3) = -\frac{1}{2}(3)^2 + 2(3) - 1$
$= \frac{1}{2}A$

$$i_L(t) = \frac{1}{2}\left[\tau^2 - 8\tau\right]_3^t + \frac{1}{2}$$

$$= \frac{1}{2}(t^2 - 8t - 9 + 24) + \frac{1}{2}$$

$$\boxed{= \frac{1}{2}t^2 - 4t + 8}$$

Approximate sketch:



(b) Power in the inductor: $p_L(t) = v_L(t)\,i_L(t)$

$\underline{0 \le t \le 1}$: $p_L(t) = \left(\frac{1}{2}t^2\right)(2t) = \boxed{-t^3}$

$\underline{1 \le t \le 3}$: $p_L(t) = (-2t + 4)\left(-\frac{1}{2}t^2 + 2t - 1\right)$
$= t^3 - 4t^2 + 2t - 2t^2 + 8t - 4$

$$\boxed{= t^3 - 6t^2 + 10t - 4}$$

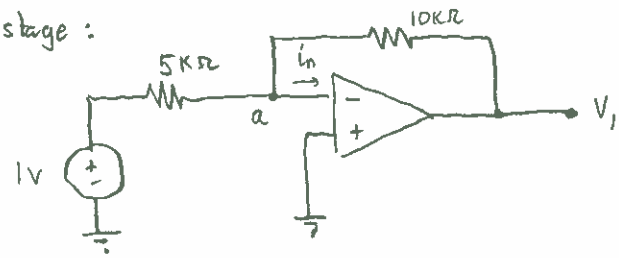
$\underline{3 \le t \le 4}$: $p_L(t) = (2t - 8)\left(\frac{1}{2}t^2 - 4t + 8\right)$
$= t^3 - 8t^2 + 16t - 4t^2 + 32t - 64$

$$\boxed{= t^3 - 12t^2 + 48t - 64}$$

Question 4

The op-amp circuit is in distinct stages.

(a) First stage :



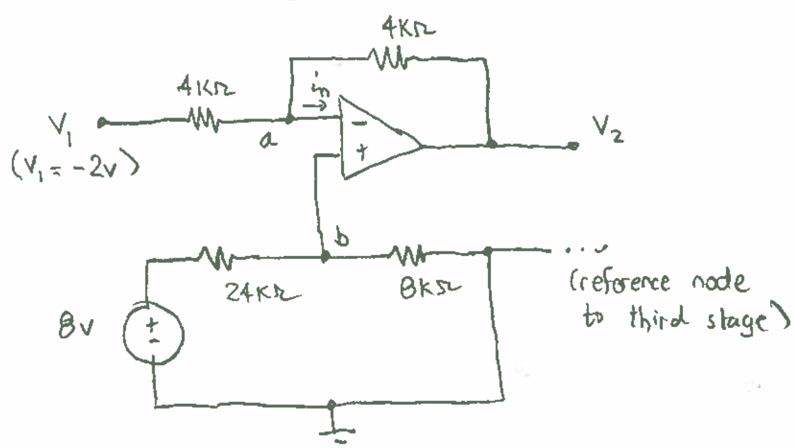
This is a simple non-inverting amplifier configuration.

Node a : $\dfrac{V_a - 1}{5K} + \dfrac{V_a - V_1}{10K} + \cancel{i_n}^{\,0} = 0$

and $V_a = 0$, so $\dfrac{-1}{5K} + \dfrac{-V_1}{10K} = 0$

$$\boxed{V_1 = -2v}$$

(b) Next stage



We may form two equations :

Node a: $\dfrac{V_a - V_1}{4K} + \dfrac{V_a - V_2}{4K} + \cancel{i_n}^{\,0} = 0$

(x 4K) $V_a - V_1 + V_a - V_2 = 0$

$V_2 = 2V_a - V_1$ (1)

Node b: $\dfrac{V_b - 8}{24K} + \dfrac{V_b}{8K} = 0$

$(\times 24K) \quad V_b - 8 + 3V_b = 0$

$4V_b = 8$

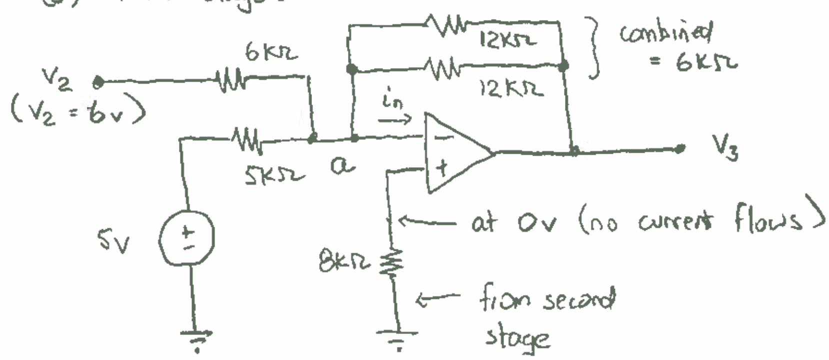
so $V_b = 2$

We also observe that $V_a = V_b$, so substitute into (1)

$V_2 = 2(2) - (-2)$

$\boxed{V_2 = 6v}$

(c) Final stage:



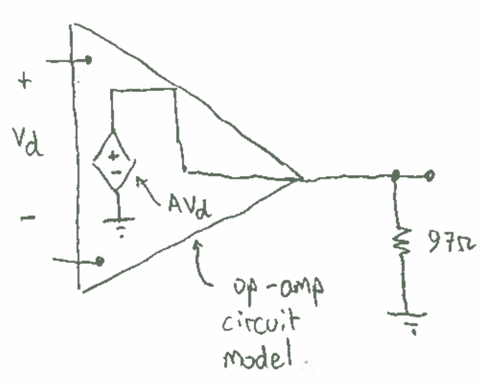
At node a: $\dfrac{V_a - V_2}{6K} + \dfrac{V_a - 5}{5K} + \dfrac{V_a - V_3}{6K} = 0$

$(\times 30k) \quad 5V_a - 5V_2 + 6V_a - 30 + 5V_a - 5V_3 = 0$

Using $V_a = 0$ and $V_2 = 6v$,

$-30 - 30 - 5V_3 = 0$, so $\boxed{V_3 = -12v}$

(d) Adding the 97Ω resistor at V_3 <u>has no effect</u>.

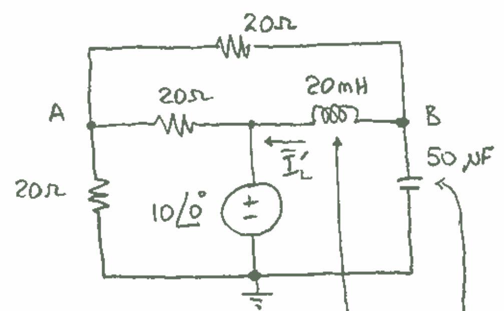


Placing a resistor in parallel with voltage source does not affect the voltage (only the current)

Question 5

We are required to solve the circuit by superposition.

(i) Voltage source acting alone: $v(t) = 10\cos(1000t)$ V.
Zeroing the current source gives:



$Z_L = j\omega L = j(1000)(0.02) = j20\,\Omega$

$Z_C = 1/j\omega C = \dfrac{1}{j(10^3)(50\times10^{-6})} = -j20\,\Omega$

Using the node-voltage method:

Node A: $\dfrac{\bar{V}_A - 10}{20} + \dfrac{\bar{V}_A}{20} + \dfrac{\bar{V}_A - \bar{V}_B}{20} = 0$

(x20) $\bar{V}_A - 10 + \bar{V}_A + \bar{V}_A - \bar{V}_B = 0$

$$3\bar{V}_A - \bar{V}_B = 10 \qquad (1)$$

Node B: $\dfrac{\bar{V}_B - 10}{j20} + \dfrac{\bar{V}_B}{-j20} + \dfrac{\bar{V}_B - \bar{V}_A}{20} = 0$

(xj20) $\bar{V}_B - 10 - \bar{V}_B + j(\bar{V}_B - \bar{V}_A) = 0$

$$j(\bar{V}_B - \bar{V}_A) = 10 \qquad (2)$$

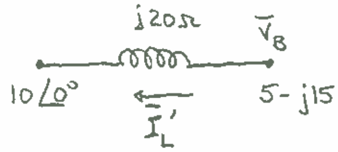
Let's solve for $\bar{V}_B$. From (1), $V_A = \frac{1}{3}(10 + \bar{V}_B)$. Substitute into (2)

$$\bar{V}_B - \tfrac{1}{3}(10 + \bar{V}_B) = \frac{10}{j}$$

(x3) $3\bar{V}_B - 10 - \bar{V}_B = -j30$

$2V_B = 10 - j30$

so $\bar{V}_B = 5 - j15$



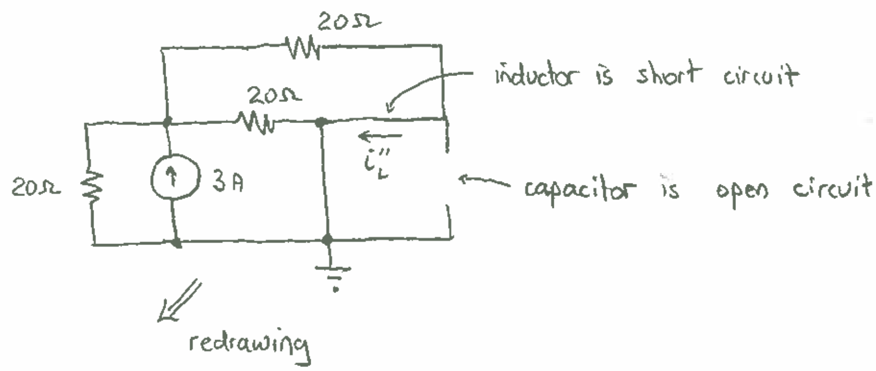
$$\bar{I}_L' = \frac{\bar{V}_B - 10}{j20} = \frac{5 - j15 - 10}{j20}$$

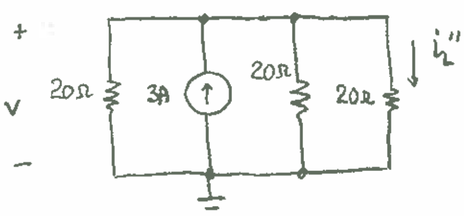
$$= \frac{-5}{j20} - \frac{j15}{j20} = -0.75 + j0.25$$

$$= 0.7906 \angle 161.57°$$

so $i_L'(t) = 0.7906 \cos(1000t + 161.57°)$

(ii) Current source acting alone : $i = 3$ A (DC source, $\omega = 0$)
Zeroing the voltage source :





3 resistors in parallel

$$R_{eq} = \left[\frac{1}{20} + \frac{1}{20} + \frac{1}{20}\right]^{-1} = \frac{20}{3}\,\Omega$$

Therefore, $V = 3\text{ A} \times R_{eq}$
$= 20$ V.

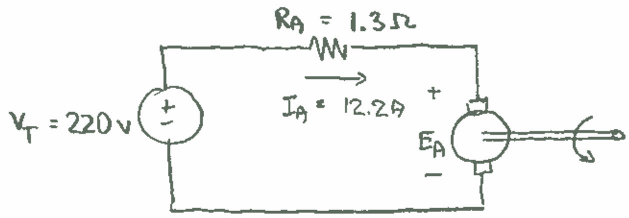
And $i_L'' = \dfrac{V}{20\Omega} = 1$ A

Finally, by superposition

$$\boxed{i_L(t) = 1 + 0.7906 \cos(1000t + 161.57°)}$$

Question 6

The armature circuit for the motor is given by



$P_{out} = 3HP$
$n_m = 950$ rpm

(a) First, $\omega_m = \frac{2\pi}{60} \times n_m = \frac{2\pi}{60} \times 950 = 99.484$ rads/sec

To find P_{dev}, we need E_A, so we can use the KVL equation

$$-V_T + I_A R_A + E_A = 0$$
$$-200 + (12.2)(1.3) + E_A = 0$$
$$\therefore \quad E_A = 204.14 \text{ V}$$

Then $P_{dev} = I_A E_A = (12.2)(204.14) = 2490.51$ W

From this, we may determine $T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{2490.51}{99.484} = 25.034$ Nm

The power lost (absorbed) in R_A is $P_A = I_A^2 R_A$
$= (12.2)^2(1.3)$
$= 193.49$ W

The rotational losses : $P_{rot} = P_{dev} - P_{rot}$
$= 2490.51 - 3 \times 746$
$= 252.51$ W

(b) For this, we will need the machine constant

$$K\phi = \frac{E_A}{\omega_m} = \frac{204.14}{99.484} = 2.052$$

$\boxed{\text{OR}}$ $K\phi = \frac{T_{dev}}{I_A} = \frac{25.034}{12.2} = 2.052$

Given $P_{rot} = 252.51$ W, $T_{rot} = \frac{P_{rot}}{\omega_m} = 2.538$ Nm

With the machine now disconnected from the mechanical load, the only work that the machine is doing is to overcome frictional losses. We should expect the motor's speed now to increase relative to part (a).

$$T_{rot} = 2.538 \text{ Nm} \quad \text{(independent of speed)}$$

Therefore, $T_{dev} = T_{rot} = 2.538$ Nm

The new armature current is $I_A = \dfrac{T_{dev}}{K\phi} = \dfrac{2.538}{2.052} = 1.237$ A

This will give a different value of E_A from which we can calculate n_m.

$$E_A = V_T - I_A R_A$$
$$= 220 - (1.237)(1.3) = 218.39 \text{ V}$$

$$\omega_m = \frac{E_A}{K\phi} = \frac{218.39}{2.052} = 106.43 \text{ rads/sec}$$

In rpm, $n_m = \dfrac{60}{2\pi} \times \omega_m = \boxed{1016.3 \text{ rpm}}$