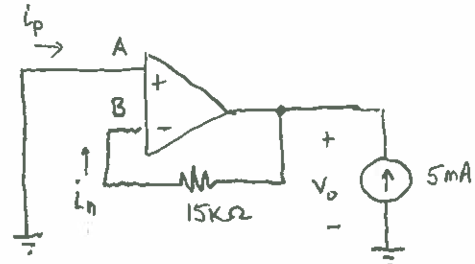
Problem 1

(a)



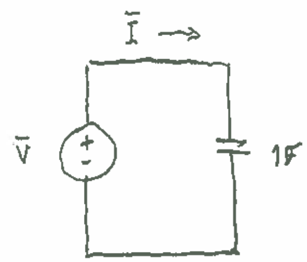
At node A, $V_A = 0$

And $V_B = 0$

At node B, $\frac{V_B - V_o}{15K} - \cancel{i_n} = 0$, so $V_o = V_B$

Therefore, $\boxed{V_o = 0}$

(b)



where $v(t) = \sin(t)$
$= \cos(t - 90°)$

and $\bar{V} = 1\angle -90°$

For the capacitor, $Z_c = \frac{1}{j\omega C}$ where $\omega = 1$ rad/sec, $C = 1$ F

so $Z_c = -j\ \Omega$

Therefore, $\bar{I} = \frac{\bar{V}}{Z_c} = \frac{1\angle -90°}{-j} = \frac{1\angle -90°}{1\angle -90°}$

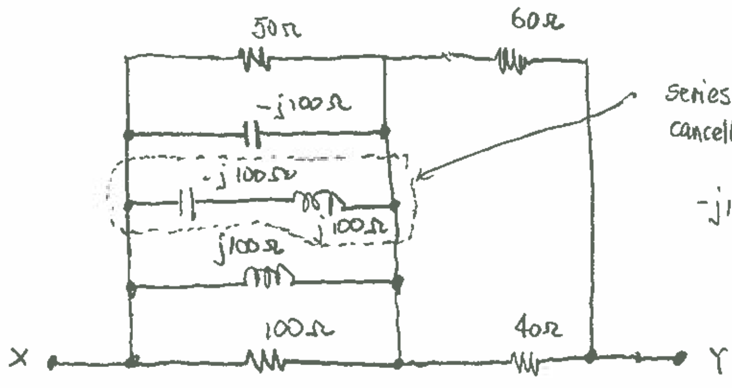
$\boxed{\bar{I} = 1\text{ A}}$

(c) We have $\omega = 100$ rad/sec, so the impedances of the inductors and capacitors will be

$$Z_L = j\omega L = j(100)\times 1 = +j100\Omega$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j(100)(100\times 10^{-6})} = -j100\Omega$$

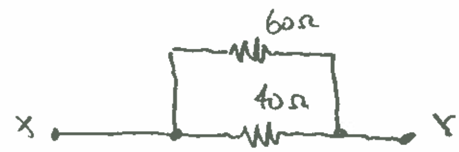
The circuit is :



series impedance cancellation!

$-j100\Omega + j100\Omega = 0\Omega$

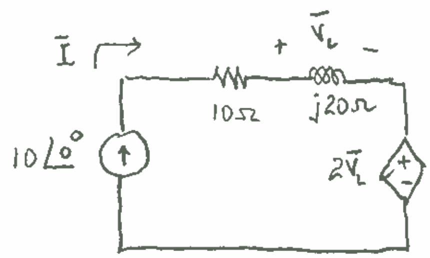
The circuit simplifies simply to



$$Z_{eq} = \frac{60 \times 40}{60 + 40}$$

$$\boxed{Z_{eq} = 24\Omega}$$

(d)



We know $\bar{V}_L = \bar{I}\,(j20)$

so $\bar{V}_L = 10\angle 0^\circ\,(j20)$

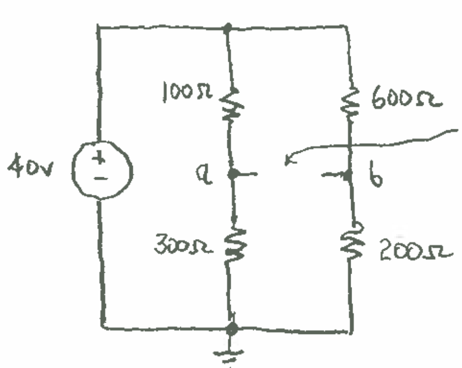
$\bar{V}_L = 200\angle 90^\circ$

Complex power $\bar{S} = \frac{1}{2}\bar{V}\bar{I}^*$, where $\bar{V} = 2\bar{V}_L = 400\angle 90^\circ$

$\bar{S} = \frac{1}{2}\left(400\angle 90^\circ\right)\left(10\angle 0^\circ\right)$

$$\boxed{\bar{S} = 2000\angle 90^\circ = j2000 \text{ VA}}$$

(e)



Capacitor behaves as an open circuit at D.C.

We need the voltage V_{ab} to determine the energy.

We have two voltage dividers

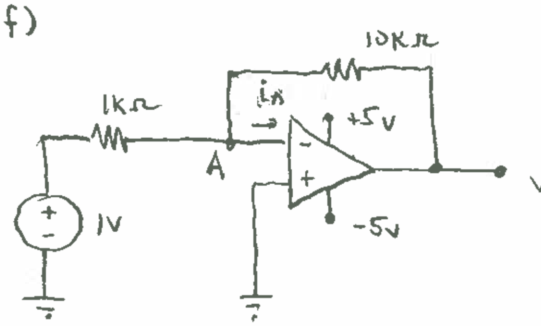
$$V_a = \frac{300}{300+100} \times 40 = 30\,\text{v} \qquad V_b = \frac{200}{200+600} \times 40 = 10\,\text{v}$$

so $V_{ab} = 30 - 10 = 20$ v.

Energy in the capacitor: $W = \frac{1}{2}Cv^2 = \frac{1}{2}(0.1)(20^2)$

$$\boxed{W = 20\text{ J}}$$

(f)



At node A,

$$\frac{V_A - 1}{1K\Omega} + \frac{V_A - V_o}{10K\Omega} - \cancel{i_n} = 0$$

and $V_A = 0$

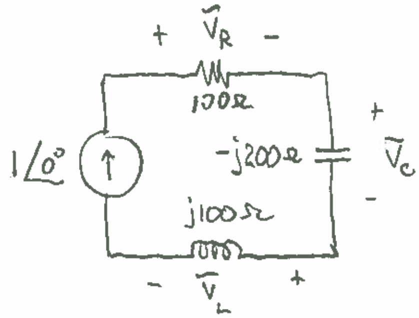
$$\frac{0-1}{1000} + \frac{0 - V_o}{10000} = 0$$

From this, $V_o = -10$ v.

However, the op-amp is only supplied ± 5v, so it will be driven into saturation.

so $\boxed{V_o = -5\,\text{v}}$

(g)



The voltages are

$$\bar{V}_R = 100\angle 0°\ \text{v}$$

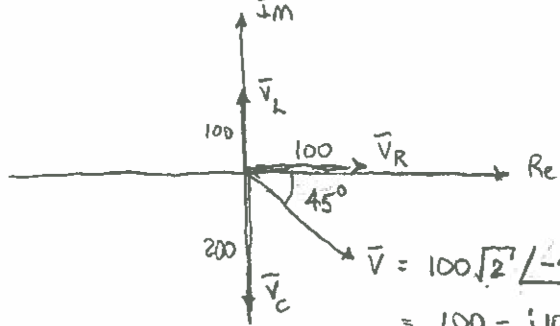
$$\bar{V}_c = -j200 = 200\angle -90°\ \text{v}$$

$$\bar{V}_L = j100 = 100\angle 90°\ \text{v}$$

And the total voltage $\bar{V} = \bar{V}_R + \bar{V}_c + \bar{V}_L$

$$= 100 - j200 + j100 = 100 - j100\ \text{v}.$$

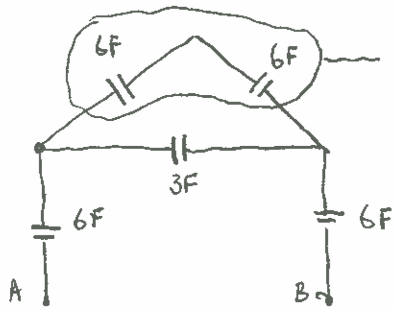
Phasor diagram



$\bar{V} = 100\sqrt{2}\angle{-45^\circ}$

$= 100 - j100$ V.

(h)

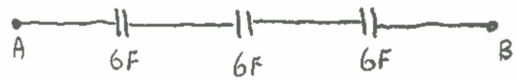


series capacitors $\dfrac{6\times 6}{6+6} = 3F$

→ and parallel with 3F

$3F + 3F = 6F.$

This simplifies to



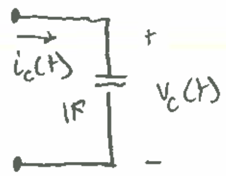
Three series capacitors $C_{AB} = \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right)^{-1}$

$= \left(\frac{3}{6}\right)^{-1}$

$\boxed{C_{AB} = 2F}$

Problem 2

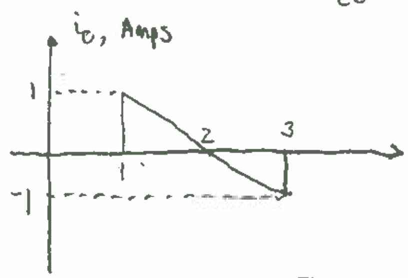
(a)



For the capacitor, $i_c(t) = \frac{1\, dv_c(t)}{dt}$

We are given a waveform for $i_c(t)$, so solve for $v_c(t)$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(\tau)\,d\tau + \cancel{v(0)}^{\,0}$$



This line has slope $\frac{1-(-1)\ \text{A}}{1-3\ \text{secs}} = -1$

The equation of the line between $t=1$ and $t=3$

$$i_c(t) = -t + i_o$$

The intercept i_o can be calculated from any point on the line. At $t=2$, $i_c(t) = 0$

$$i_c(t) = 0 = -2 + i_o, \quad \text{so} \quad i_o = 2\ \text{A}.$$

Therefore, $i_c(t) = -t + 2$.

The voltage on the capacitor by interval:

$t < 1$ s: $\boxed{v_c(t) = 0}$

$1 \le t \le 3$: $v_c(t) = \frac{1}{C}\int_1^t (-\tau + 2)\,d\tau = \left. -\frac{\tau^2}{2} + 2\tau \right|_1^t$

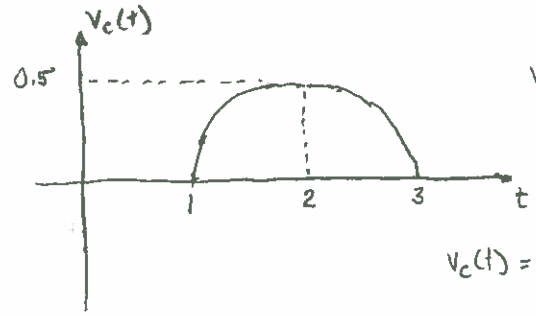
$$v_c(t) = -\frac{t^2}{2} + 2t - \left(-\frac{1}{2} + 2\right)$$

so $\boxed{v_c(t) = -\frac{t^2}{2} + 2t - 1.5 \ \text{v.}}$

$t > 3$: Evaluating the above equation at $t = 3$,

$$v_c(3) = -\frac{(3)^2}{2} + 2(3) - 1.5 = -4.5 + 6 - 1.5$$

so $\boxed{v_c(t) = 0\ \text{v}}$



$$V_c(2) = -\frac{2^2}{2} + 2(2) - 1.5$$
$$= 0.5 \text{ v.}$$

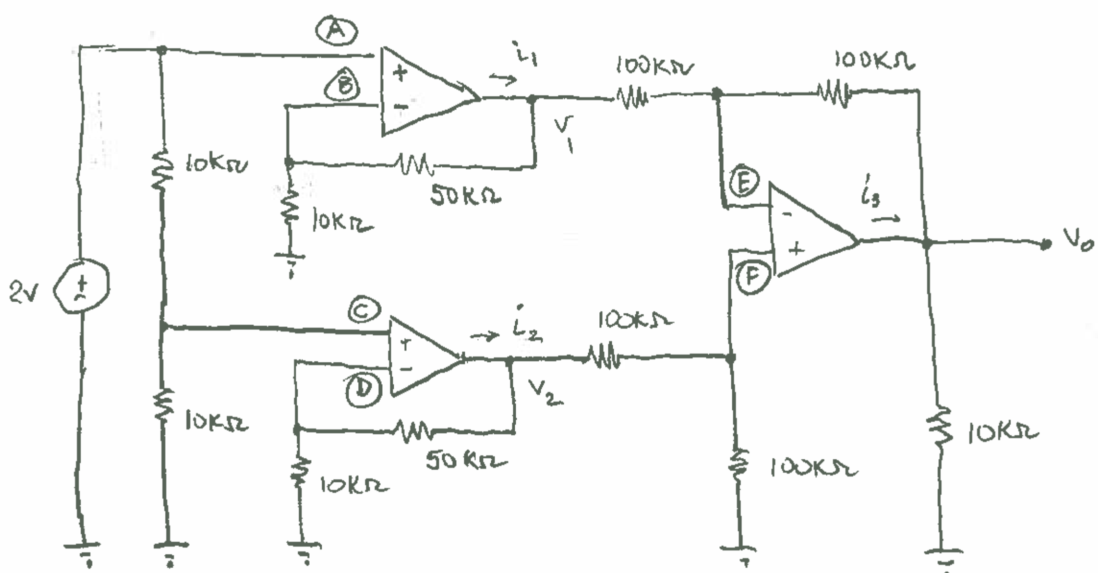
$V_c(t) = 0$ for $0 \le t < 1$,
$3 \le t < 4$.

(b) Power in capacitor: $P_c(t) = V_c(t)\, i_c(t)$

$$P_c(t) = \left(-\frac{t^2}{2} + 2t - 1.5\right)\left(-t + 2\right)$$
$$= \frac{t^3}{2} - 2t^2 + 1.5t - \frac{2t^2}{2} + 4t - 3$$

$$\boxed{P_c(t) = \frac{t^3}{2} - 3t^2 + 5.5t - 3 \quad \text{W.}}$$

Problem 3



(a) At node A, we know $V_A = 2$v, so $V_B = 2$v.

At node B: $\frac{V_B}{10K} + \frac{V_B - V_1}{50K} = 0$

so $\frac{2}{10K} + \frac{2 - V_1}{50K} = 0$

giving $10 + 2 - V_1 = 0$

$$\boxed{V_1 = 12\text{ v}}$$

At node C: $\dfrac{V_C}{10K} + \dfrac{V_C - 2}{10K} = 0$

$V_C + V_C - 2 = 0$, so $V_C = 1$ v.

At node D: $\dfrac{V_D}{10K} + \dfrac{V_D - V_2}{50K} = 0$, where $V_D = V_C = 1$ v.

$$\frac{1}{10K} + \frac{1 - V_2}{50K} = 0$$

giving $5 + 1 - V_2 = 0$

$$\boxed{V_2 = 6\text{ V}}$$

(b) At node E: $\dfrac{V_E - V_1}{100K} + \dfrac{V_E - V_o}{100K} = 0$

$V_E - V_1 + V_E - V_o = 0$

so $2V_E = V_1 + V_o$

$V_o = -V_1 + 2V_E$

At node F: $\dfrac{V_F}{100K} + \dfrac{V_F - V_2}{100K} = 0$, where $V_F = V_E$

$V_F + V_F - V_2 = 0$

$V_F = \frac{1}{2} V_2$

so $V_F = \frac{1}{2}(6) = 3$ v.

Therefore

$V_o = -V_1 + 2V_F = -12 + 2(3)$

$$\boxed{V_o = -6\text{ v}}$$

(c) At node V_1: $-i_1 + \dfrac{V_1 - V_B}{50K} + \dfrac{V_1 - V_E}{100K} = 0$

We know $V_1 = 12v$, $V_B = 2V$, $V_E = 3v$

$$-i_1 + \frac{12-2}{50K} + \frac{12-3}{100K} = 0$$

$$-(100K)i_1 + 2(12-2) + 12-3 = 0$$

$$-(100K)i_1 + 20 + 9 = 0$$

$$\boxed{i_1 = \frac{29}{100K} = 0.29\text{ mA}}$$

At node V_2: $-i_2 + \dfrac{V_2 - V_F}{100K} + \dfrac{V_2 - V_D}{50K} = 0$

We know $V_2 = 6v$, $V_D = 1v$, $V_F = 3V$.

$$-i_2 + \frac{6-3}{100K} + \frac{6-1}{50K} = 0$$

$$-(100K)i_2 + 6-3 + 2(6-1) = 0$$

$$(-100K)i_2 + 3 + 10 = 0$$

$$\boxed{i_2 = \frac{13}{100K} = 0.13\text{ mA}}$$

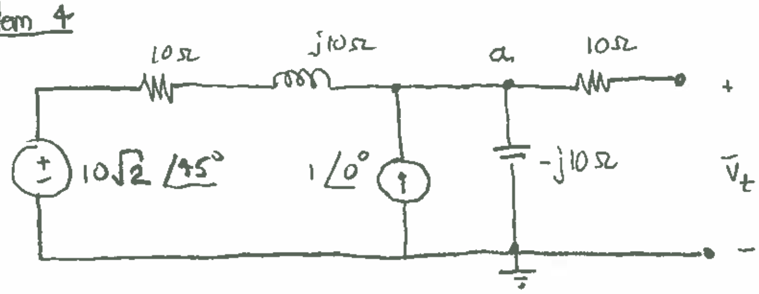
At node V_0: $-i_3 + \dfrac{V_0 - V_E}{100K} + \dfrac{V_0}{10K} = 0$

We know $V_0 = -6v$, $V_E = 3v$

$$-(100K)i_3 + -6-3 + 10(6)$$

$$\boxed{i_3 = \frac{63}{100K} = -0.51\text{ mA}}$$

Problem 4



(a) With the circuit labelled as shown, $\bar{V}_t = \bar{V}_a$

At node a: $$\frac{\bar{V}_a - 10\sqrt{2}\angle 45°}{10+j10} - 1 + \frac{\bar{V}_a}{-j10} = 0$$

where $10\sqrt{2}\angle 45° = 10\sqrt{2}\cos(45°) + j10\sqrt{2}\sin(45°)$
$= 10 + j10$

so $$\frac{\bar{V}_a - 10 - j10}{10+j10} - 1 + \frac{\bar{V}_a}{-j10} = 0$$

$$\bar{V}_a\left(\frac{1}{10+j10} + \frac{1}{-j10}\right) = \frac{10+j10}{10+j10} + 1$$

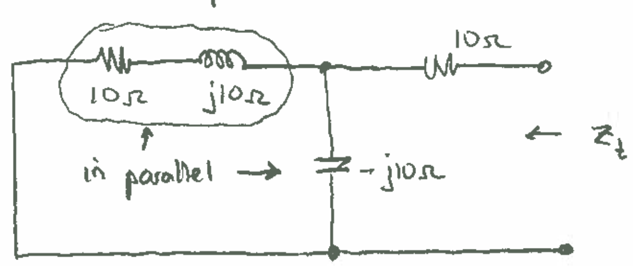
$$\bar{V}_a\left(\frac{-j10 + 10 + j10}{(-j10)(10+j10)}\right) = 2$$

$$\bar{V}_a\left(\frac{10}{-j100 + 100}\right) = 2$$

so $\bar{V}_a = 2(10 - j10)$
$= 20 - j20$

Finally, $\boxed{\bar{V}_t = \bar{V}_a = 20 - j20 = 20\sqrt{2}\angle -45°}$

(b) There are no dependent sources, so we can zero the sources



We have $Z_t = \dfrac{(10+j10)(-j10)}{(10+j10-j10)} + 10\,\Omega$

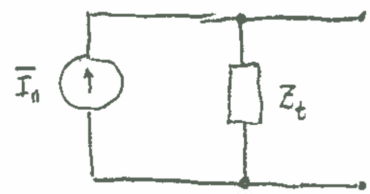
$$= \frac{-j100 + 100}{10} + 10\,\Omega$$

$$Z_t = 10 - j10 + 10$$

so $\boxed{Z_t = 20 - j10\,\Omega = 22.36\angle{-26.57^\circ}}$

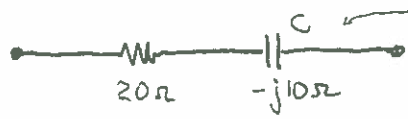
(c) The Norton current is

$$\bar{I}_n = \frac{\bar{V}_t}{Z_t} = \frac{20\sqrt{2}\angle 45^\circ}{22.36\angle{-26.57^\circ}} \qquad \boxed{\begin{aligned} &= 1.265\angle 71.57^\circ \\ &= 0.396 + j1.2 \end{aligned}}$$



NORTON EQUIVALENT

(d) We have $Z_t = 20 - j10\,\Omega$



capacitor because of negative reactance.

$$Z_L = -j10 = \frac{1}{j\omega C}, \qquad \text{where } \omega = 100 \text{ rad/sec}$$

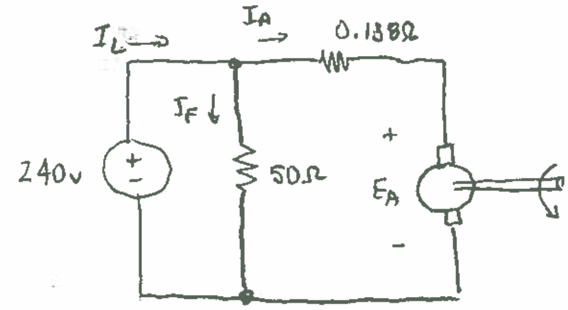
$$\text{so} \quad \omega C = \frac{1}{10}, \qquad C = \frac{1}{10\,\omega}\ \text{F}$$

$$C = \frac{1}{10(100)}\ \text{F}$$

$$\boxed{R = 20\,\Omega,\ C = 1000\,\mu\text{F}}$$

Problem 5

(a)



$n_m = 1200$ rpm
$P_{out} = P_{dev} = 25$ HP

Initially, $n_m = 0$, so the induced voltage E_A

$$E_A = K\phi \underbrace{\omega_m}_{=0} = 0$$

Therefore, the initial armature current $I_A = \dfrac{V_T}{R_A}$

$$\boxed{I_A = \frac{240}{0.138} = 1739.1\ \Omega}$$

(b) Since there are no rotational losses, $P_{dev} = P_{out}$

$$P_{out} = 25 \times 746 = 18,650\ \text{W}.$$

The output torque $T_{out} = T_{dev}$

$$T_{out} = \frac{P_{out}}{\omega_m} = \frac{18,650}{1200 \times \frac{2\pi}{60}} = 148.412\ \text{N-m}$$

We are given the efficiency $\eta = 90\%$, so total power input P_{in}

$$P_{in} = \frac{P_{out}}{\eta} = \frac{18,650}{0.9} = 20,722.2\ \text{W}$$

This means that $P_{in} = V_T I_L$, so $I_L = \dfrac{20,722.2}{240}$

$$I_L = 86.34\ \text{A}$$

KCL gives $I_L = I_F + I_A = \left(\dfrac{V_T}{R_F}\right) + I_A$

so $I_A = 86.34 - \frac{240}{50} = 81.54\ A$

Armature and field losses :

$$P_F = \frac{V_T^2}{R_F} = \frac{(240)^2}{50} = 1152\ W$$

$$P_A = I_A^2 R_A = (81.54)^2 (0.138) = 917.53\ W$$

Back EMF : $E_A = V_T - I_A R_A$
$= 240 - (81.54)(0.138) = 228.7\ V$

Summary :

$$\boxed{\begin{aligned} T_{out} &= 148.412\ N\text{-}m \\ P_A &= 917.53\ W \\ P_F &= 1152\ W \\ E_A &= 228.7\ V \end{aligned}}$$

(c) From the above information, we can find the machine constant.

$$K\phi = \frac{T_{dev}}{I_A} = \frac{148.412}{81.54} = 1.82$$

with $T_{dev} = 100\ N\text{-}m$, the new I_A is

$$I_A = \frac{T_{dev}}{K\phi} = \frac{100}{1.82} = 54.94\ A$$

Then $E_A = V_T - I_A R_A = 240 - (54.94)(0.138)$
$= 232.4\ V$

The new speed therefore :

$$\omega_m = \frac{E_A}{K\phi} = \frac{232.4}{1.82} = 127.7\ rads/sec$$

$$n_m = \omega_m \times \frac{60}{2\pi} = 127.7 \times \frac{60}{2\pi}$$

$$\boxed{n_m = 1219\ rpm}$$