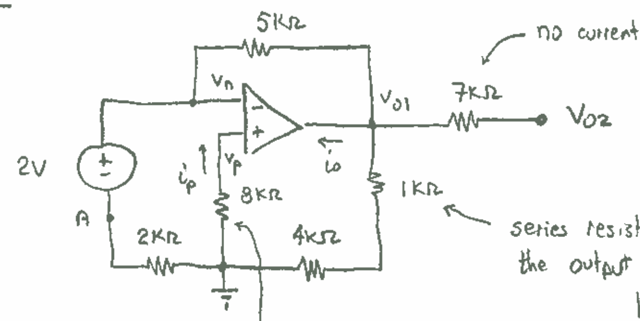
Question 1

(a)



no current flows.

series resistance of 5kΩ at the output of the opamp has <u>no effect</u> on V_o.

no current flows, so $V_p = 0$; therefore, $V_n = 0$.

One way to solve this is by recognizing a <u>supernode</u> between nodes V_n and A

Supernode : $\frac{V_A}{2k} + \frac{V_n - V_{o1}}{5k} + \cancel{i_n}^{\,0} = 0$, where $V_n = 0$

$(\times 5k)$ $\quad 2.5\, V_A + 0 - V_{o1} = 0$

$V_{o2} = 2.5\, V_A$

Supernode dependence : $\quad V_n - V_A = 2$

$0 - V_A = 2 \qquad \therefore \; V_A = -2$

therefore, $\quad V_{o1} = -5V.$

Another way is to write a combined equation at node V_n

$$\frac{V_n - 2}{2K} + \frac{V_n - V_{o1}}{5K} + \cancel{i_n}^{\,0} = 0 \quad \Rightarrow \quad \frac{-2}{2k} - \frac{V_{o1}}{5k} = 0$$

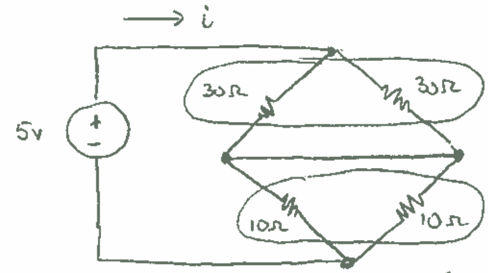
so $\boxed{V_{o1} = -5V}$

Since there is no current in the output 7kΩ resistor, $\boxed{V_{o2} = V_{o1} = -5V}$

At node V_{o1}, $\quad \frac{V_{o1} - V_n}{5K} + \frac{V_{o1}}{5K} + i_o = 0$

$$\frac{-5 + -5}{5k} = -i_o \quad , \text{ so } \boxed{i_o = 2\,mA}$$

(b) Since the source is DC, the inductor is a <u>short circuit</u> and the capacitor is an <u>open circuit</u>. Redrawing ...



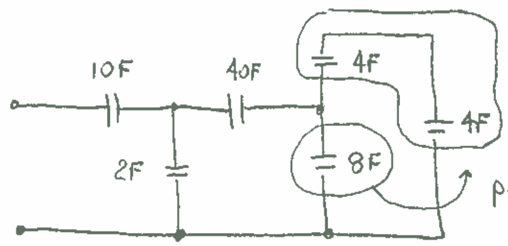
parallel = 15Ω

parallel = 5Ω

series 15Ω + 5Ω = 20Ω

$$\boxed{i = \frac{5V}{20\Omega} = 0.25A}$$

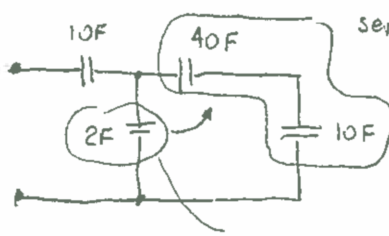
(c)



series : 2F

parallel 8F + 2F = 10 F

Redrawing ...



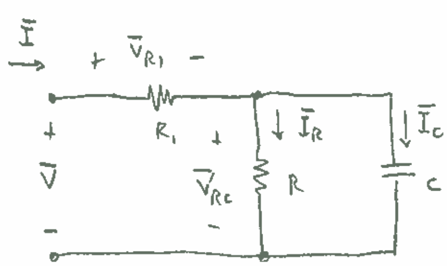
series = $\frac{40 \times 10}{40+10}$ = 8F

parallel 8F + 2F = 10 F

Finally, 10 F in series with 10 F

$$\boxed{C_{eq} = 5\,F}$$

(d)



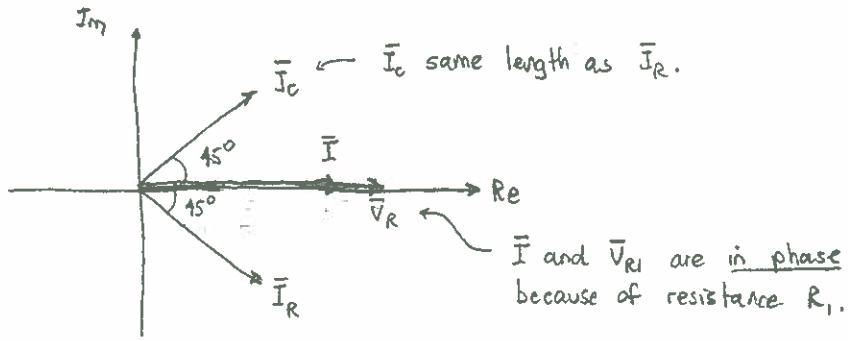
Given $|\bar{I}_R| = |\bar{I}_c|$

Current $\bar{I}_R = \frac{\bar{V}_{Rc}}{R}$ and $\bar{I}_c = \frac{\bar{V}_{Rc}}{1/j\omega c} = (j\omega c)\bar{V}_{Rc}$

So $\bar{I}_c$ <u>leads</u> $\bar{I}_R$ by $90°$

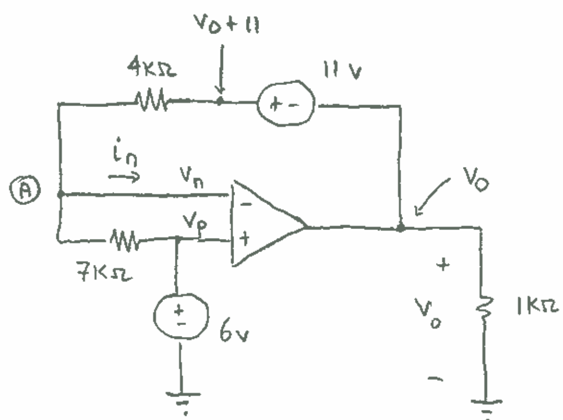
Since we are told that the angle of $\bar{V}_{R1}$ is 0°, this is also the angle of $\bar{I}$.

The imaginary components of $\bar{I}_R$ and $\bar{I}_c$ must therefore <u>cancel</u>.



$\bar{I} = \bar{I}_R + \bar{I}_c$

(e)



Because $V_p = 6$, V_n must also be 6 v. Then, at node A,

$$\frac{V_A - V_p}{7K} + \cancel{i_n}^{\,0} + \frac{V_A - (V_o + 11)}{4K} = 0$$

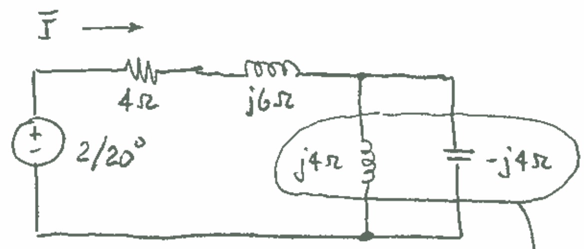
where $V_A = V_p = V_n = 6v$

so $$\frac{V_A - (V_o + 11)}{4K} = 0$$

$$V_o = V_a - 11 = 6 - 11$$

$$\boxed{V_o = -5\text{ v}}$$

(f)



$Z_{eq} = \dfrac{j4 \times -j4}{j4 - j4} = \infty\ \Omega$

Open circuit!

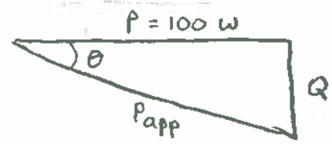
Therefore, $\boxed{\bar{I} = 0}$.

(g)

- Power factor = 0.6 leading
- P = 100 W

Current leads voltage, suggesting a capacitive load because $\theta_I > \theta_V$. Therefore, $\theta = \theta_V - \theta_I$ is negative.

Power triangle

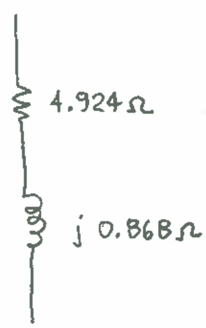


$P_{app} = \dfrac{100}{\cos(\theta)} = \dfrac{100}{0.6} = \boxed{166.67\ VA}$

(h) We know $\bar{V} = 10\angle{-20°}$ and $\bar{I} = 2\angle{-30°}$

so $Z = \dfrac{\bar{V}}{\bar{I}} = 5\angle 10°$, or $Z = 4.924 + j0.868\ \Omega$

This is a resistor and inductor in series



← $Z_L = j\omega L$, so $L = \dfrac{Z_L}{j\omega}$

$L = \dfrac{0.868}{100\pi}$

$\boxed{L = 2.76\ mH}$

Question 2

(a) For the inductor, $v(t) = L\dfrac{di(t)}{dt}$

Interval $0 \le t \le 1$ second: $i(t) = 0$, so $v(t) = 0$

For $1 \le t \le 3$, $i(t)$ is a straight line with slope

$$\text{slope} = \frac{10\text{ A}}{2\text{ secs}} = 5$$

$$\text{so} \quad V(t) = L\frac{di(t)}{dt} = 4 \times 5 = 20\text{ V}$$

For $t > 3$, $i(t)$ is a flat line at 10 A, with zero slope

$$V(t) = 4 \times 0 = 0\text{ v}$$

For the capacitor, $i(t) = C\dfrac{dv(t)}{dt}$, so $V(t) = \dfrac{1}{C}\displaystyle\int_{t_1}^{t_2} i(t)\,dt + v(t_1)$

For $0 \le t \le 1$, $i(t) = 0$,

$$\text{so} \quad v(t) = \frac{1}{2}\int_0^1 0\,dt + v(0) = -1\text{v}$$

For $1 \le t \le 3$, we need the equation of the line.

$$i(t) = \underbrace{5t}_{\text{slope}} - \underbrace{5}_{\text{intercept}}$$

$$\text{so} \quad v(t) = \int_1^t i(t)\,dt + v(1)$$

$$= \frac{1}{2}\int_1^t (5t - 5)\,dt + -1$$

$$v(t) = \frac{1}{2}\left(\frac{5}{2}t^2 - 5t\right)\Big|_1^t + -1$$

$$v(t) = \frac{5}{4}t^2 - \frac{5}{2}t - \left(\frac{5}{4} - \frac{5}{2}\right) + -1$$

$$= \frac{5}{4}t^2 - \frac{5}{2}t + \frac{1}{4}$$

For $t > 3$, $\quad i(t) = 10$, so

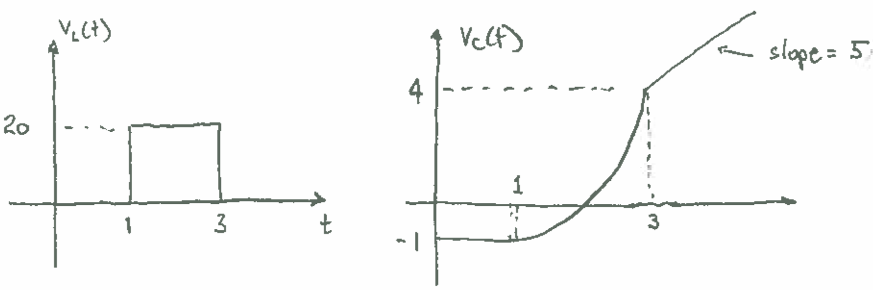
$$v(t) = \frac{1}{2}\int_3^t 10\,dt + v(3)$$

$$v(3) = \frac{5}{4}(3)^2 - \frac{5}{2}(3) + \frac{1}{4} = 4$$

$$v(t) = 5t\Big|_3^t + 4$$

$$= 5t - 15 + 4 = 5t - 11$$

Sketch:



(b) Power in the inductor: $\quad p_L(t) = v_L(t)\,i(t)$

Interval $0 \le t \le 1$: $\quad p_L(t) = 0$

$1 \le t \le 3$: $\quad p_L(t) = 20 \times (5t - 5)$
$= 100t - 100$ W

$t > 3$: $\quad p_L(t) = 0$

Power in the capacitor: $\quad p_C(t) = v_C(t)\,i(t)$

$0 \le t < 1$: $\quad p_C(t) = 0$

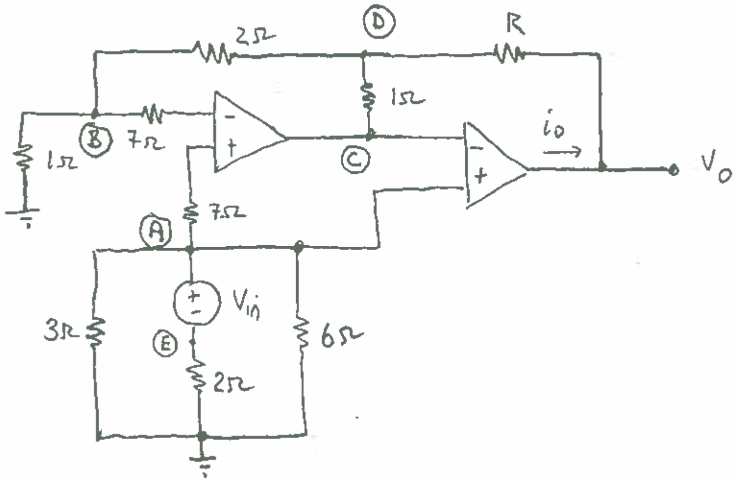
$1 \le t < 3$: $\quad p_C(t) = \left(\frac{5}{4}t^2 - \frac{5}{2}t + \frac{1}{4}\right)(5t - 5)$

$$= \frac{25}{4}t^3 - \frac{25}{2}t^2 + \frac{5}{4}t - \frac{25}{4}t^2 + \frac{25}{2}t - \frac{5}{4}$$

$$= \frac{25}{4}t^3 - \frac{75}{4}t^2 + \frac{55}{4}t - \frac{5}{4} \quad \text{W}$$

$t \ge 3$: $\quad p_C(t) = 10 \times (5t - 11) = 50t - 110$ W.

Question 3



(a) At node A, $\frac{V_A}{3} + \frac{V_A}{6} + \frac{V_E}{2} = 0$ (supernode equation)

(×6) $2V_A + V_A + 3V_E = 0$

Dependence: $V_A - V_E = V_{in}$, so $V_E = V_A - V_{in}$

and $2V_A + V_A + 3(V_A - V_{in}) = 0$

$6V_A - 3V_{in} = 0$

so $V_A = \frac{V_{in}}{2}$.

Node B: $\frac{V_B}{1} + \frac{V_B - V_D}{2} = 0$, where $V_B = V_A$

$2V_B + V_B = V_D$

so $V_D = 3V_B = 3V_A = \frac{3}{2}V_{in}$

Node D: $\frac{V_D - V_o}{R} + \frac{V_D - V_C}{1} + \frac{V_D - V_B}{2} = 0$

where $V_B = V_A$ and $V_C = V_A$

$\frac{V_D - V_o}{R} + V_D - V_A + \frac{V_D - V_A}{2} = 0$

$2(V_D - V_o) + 3R(V_D - V_A) = 0$

$2V_D - 2V_o + 3RV_D - 3RV_A = 0$

Substitute $V_D = \frac{3}{2} V_{in}$ and $V_A = \frac{1}{2} V_{in}$

$$3V_{in} - 2V_o + \frac{9}{2} R V_{in} - \frac{3}{2} R V_{in} = 0$$

$$3V_{in} + 3RV_{in} = 2V_o$$

$$\boxed{V_o = \frac{(3 + 3R) V_{in}}{2}}$$

(b) Using the answer to (a), $V_o = \frac{(3 + 3 \cdot 1) \times 6}{2}$

$$V_o = 18 \text{ V}$$

And at node V_o,

$$\frac{V_o - V_D}{R} - i_o = 0 \quad \text{where } V_D = \frac{3}{2} V_{in} = 9 \text{ V}$$

so $\frac{18 - 9}{1} - i_o = 0$

$$\boxed{i_o = 9 \text{ A}}$$

(c) The load circuit consists of just a 2Ω resistor!

V_o will not change : $V_o = 18\text{V}$

And at node V_o, $\frac{V_o - V_D}{R} - i_o + \frac{V_o}{2} = 0$

load current

$$\frac{18 - 9}{1} - i_o + \frac{18}{2} = 0$$

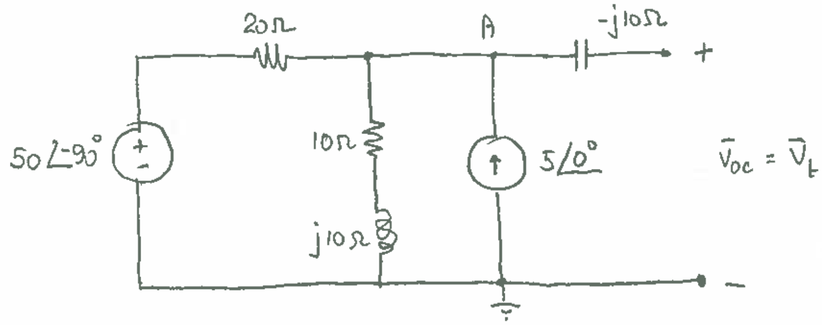
$9 + 9 = i_o$, so $\boxed{i_o = 18 \text{ A}}$

Question 4

(a) With $\omega = 1000$ r/s, $Z_L = j\omega L = j(1000)(0.01) = j10\Omega$

and $Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{j(1000)(100\times10^{-6})} = -j10\Omega$

(b)



Thevenin voltage: Open-circuit voltage will be $\bar{V}_{oc} = \bar{V}_A$

Node A: $\dfrac{\bar{V}_A - 50\angle{-90^\circ}}{20} + \dfrac{\bar{V}_A}{10+j10} - 5\angle 0^\circ = 0$

$$\frac{\bar{V}_A + j50}{20} + \frac{\bar{V}_A}{10+j10} = 5$$

(x20) $\bar{V}_A + j50 + \dfrac{2\bar{V}_A}{1+j} = 100$

(x (1+j)) $(\bar{V}_A + j50)(1+j) + 2\bar{V}_A = 100 + j100$

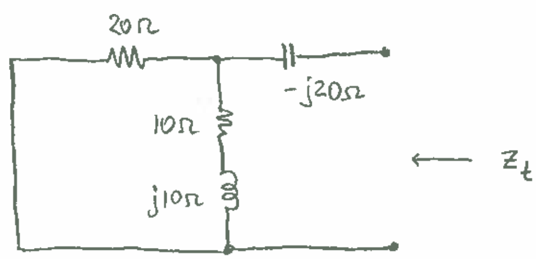
$$\bar{V}_A + j\bar{V}_A + j50 - 50 + 2\bar{V}_A = 100 + j100$$

$$\bar{V}_A(3+j) = 150 + j50$$

$$\bar{V}_A = \frac{150+j50}{3+j} = \frac{50(3+j)}{3+j}$$

$$\boxed{\bar{V}_A = 50\angle 0^\circ}$$

(c) Since there are no independent sources, we may zero the independent ones



$$Z_t = [20 \,//\, (10+j10)] - j10$$

$$= \frac{20(10+j10)}{30+j10} - j10 = \frac{200 + j200}{30+j10} - j10$$

$$Z_t = \frac{20+j20}{3+j} - j10 = \frac{(20+j20)(3-j)}{(3+j)(3-j)} - j10$$

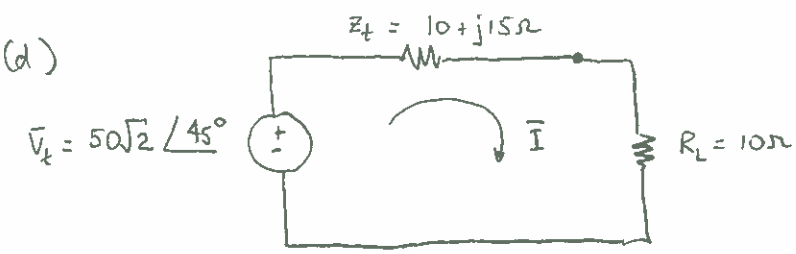
$$= \frac{60 - j20 + j60 + 20}{9 + j3 - j3 + 1} - j10$$

$$= \frac{80 + j40}{10} - j10$$

$$= 8 + j4 - j10$$

$$\boxed{\begin{aligned} Z_t &= 8 - j6 \\ &= 10\angle -36.87^\circ \end{aligned}}$$

(d)



$$\bar{I} = \frac{\bar{V}_t}{Z_t + 10} = \frac{50\sqrt{2}\angle 45^\circ}{20 + j15} = \frac{50\sqrt{2}\angle 45^\circ}{25\angle 36.87^\circ}$$

$$= 2\sqrt{2}\angle 8.13^\circ$$

from this, $I_{rms} = 2$ A

Since the load is purely resistive,

$$\boxed{\begin{aligned} P &= I_{rms}^2 \times 10 \\ &= 20\ \text{W} \end{aligned}}$$

and $\boxed{Q = 0 \text{ VAR}}$

Question 5

(a) From KVL, we know $E_A = V_T - I_A R_A$
$= 220 - (18.5)(0.5)$
$= 210.75\text{v}$

We know, $n_m = 1500$ rpm, so $\omega_m = 1500 \times \frac{2\pi}{60} = 157.08$ r/s

The machine onstant : $E_A = K\phi\omega_m$

$$K\phi = \frac{E_A}{\omega_m} = \frac{210.75}{157.08} = 1.342$$

The developed torque $T_{dev} = K\phi I_A = 1.342 \times 18.5$

$$\boxed{T_{dev} = 24.82 \text{ Nm}}$$

The total power out is $P_{out} = 5 \text{ HP} \times 746 = 3730 \text{ W}$

The developed power is $P_{dev} = T_{dev}\,\omega_m = 24.82 \times 157.08$
$= 3898.9$ W

Rotational loss : $P_{rot} = P_{dev} - P_{out} = 3898.9 - 3730$

$$\boxed{P_{rot} = 168.9 \text{ W}}$$

Armature loss : $P_A = I_A^2 R_A = (18.5)^2(0.5)$

$$\boxed{P_A = 17.11 \text{ W}}$$

Field loss : $P_F = \frac{V_F^2}{R_F} = \frac{(60)^2}{80}$

$$\boxed{P_F = 45 \text{ W}}$$

Efficiency : $P_{out} = 3730$ W. $P_{in} = P_F + V_T I_A$
$= 45 + (220)(18.5)$
$= 4115$ W

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 90.6\%$$

(b) From part (a), the rotational power loss is

$$P_{rot} = 168.9\ \text{W}$$

so that $T_{rot} = \dfrac{P_{rot}}{\omega_m} = \dfrac{168.9}{157.08} = 1.075\ \text{Nm}$

The armature current $I_A = \dfrac{T_{rot}}{K\phi} = \dfrac{1.075}{1.342} = 0.801\ \text{A}$

And the new $E_A = V_T - I_A R_A$
$= 220 - (0.801)(0.05)$
$= 219.96\ \text{V}.$

Therefore, $\omega_m = \dfrac{E_A}{K\phi} = \dfrac{219.96}{1.342} = 163.94\ \text{r/s}$

and $n_m = \omega_m \times \dfrac{60}{2\pi} = \boxed{1565.6\ \text{rpm}}$

(c) When $n_m = 0$ rpm, $I_A = \dfrac{V_T - 0}{R_A} = 440\ \text{A}$

so $T_{dev} = K\phi \times 440 = \boxed{590.34\ \text{Nm}}$