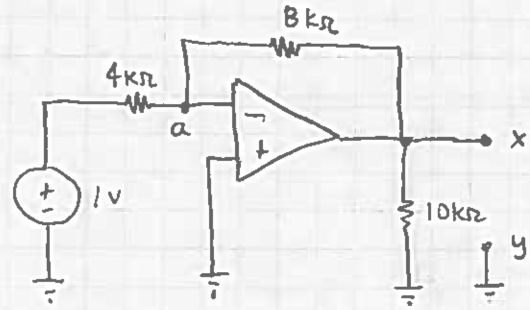
Question 1

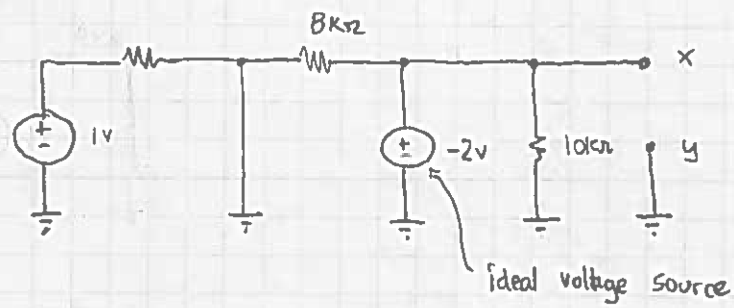
(a)



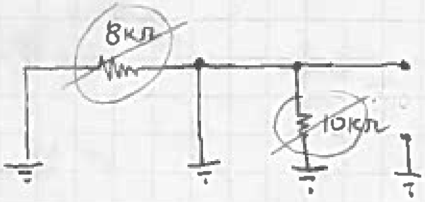
By KCL at node a, $\dfrac{V_a - 1}{4K} + \dfrac{V_a - V_x}{8K} = 0$, where $V_a = 0$

Solving gives $V_x = V_{xy} = -2V$. Thus, the op-amp is producing $-2v$ at its output. This is independent of the current being drawn at x (e.g., the 10 kΩ resistor does not affect V_x in any way).

This is equivalent to

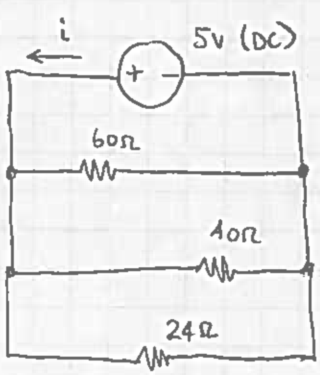


Zeroing the sources,



$R_t = 0\Omega$

(b) At DC, the impedance of the capacitor = ∞ (open circuit), and the impedance of the inductor = 0 Ω (short circuit)



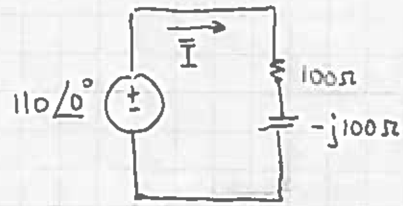
Three parallel resistors.

$$R_{eq} = \left[\frac{1}{60} + \frac{1}{40} + \frac{1}{24}\right]^{-1}$$

$$= \left[0.08333\right]^{-1} = 12\Omega$$

so $i = \dfrac{5v}{12\Omega} = 0.41667\ A$

(c)



$$\bar{I} = \frac{\bar{V}}{100 - j100} = \frac{110 \angle 0^\circ}{100 - j100}$$

$$= \frac{110}{100\sqrt{2}\angle -45^\circ} = \frac{1.1}{\sqrt{2}} \angle 45^\circ$$

In the impedance, $\bar{S} = \frac{1}{2}\bar{V}\bar{I}^* = \frac{1}{2}\left[110 \times \frac{1.1}{\sqrt{2}} \angle -45^\circ\right]$

$$= \frac{121}{2\sqrt{2}} \angle -45^\circ = \boxed{42.78 \angle -45^\circ}$$

In rectangular form, $\bar{S} = \frac{121}{2\sqrt{2}} \cos(-45^\circ) + \frac{121}{2\sqrt{2}} \sin(-45^\circ)$

$$= \boxed{30.25 - j\,30.25 \quad \text{VA}}$$

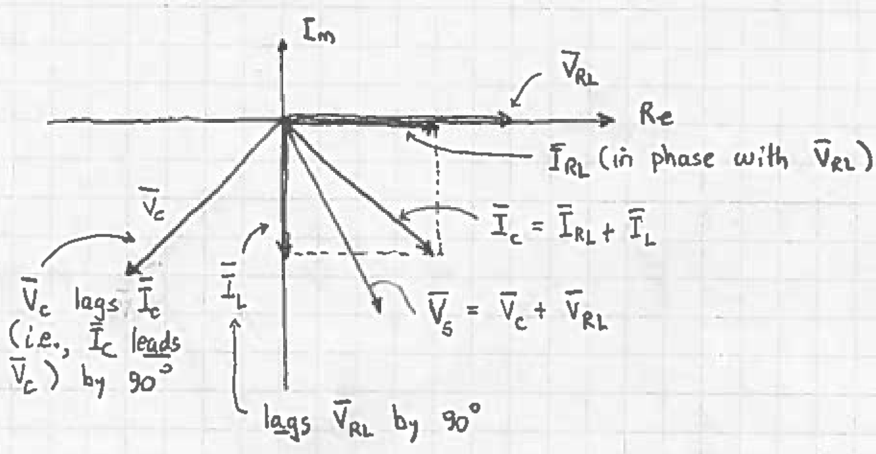
giving $P = 30.25\,\text{W}$

$Q = -30.25$ VAR

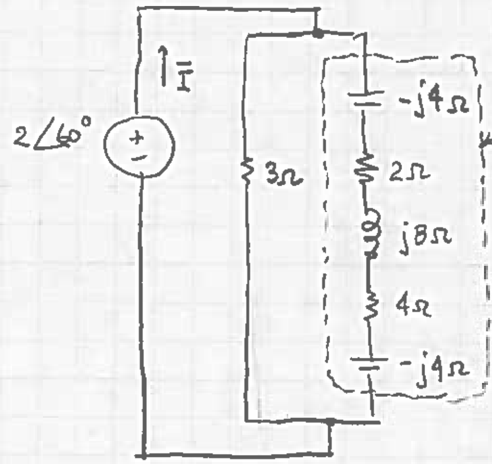
Alternatively, $P = I_{rms}^2 R = \left(\frac{1.1}{\sqrt{2}\sqrt{2}}\right)^2 \times 100 = 30.25$ W

$$Q = I_{rms}^2 X = \left(\frac{1.1}{\sqrt{2}\sqrt{2}}\right)^2 \times -100 = -30.25 \text{ VAR}$$

(d)



(e)



Impedance of series branch

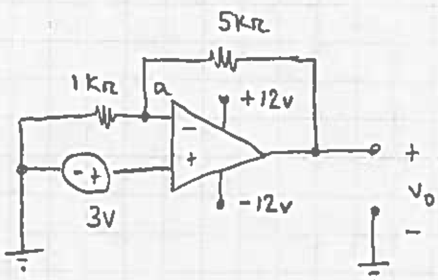
$$Z_{eq} = -j4 + 2 + j8 + 4 - j4$$
$$= 6\Omega$$

Combined with 3Ω in parallel,

$$Z_{tot} = \frac{3 \times 6}{3+6} = 2\Omega$$

Therefore $\bar{I} = \dfrac{2\angle 60^\circ}{Z_{tot}} = \dfrac{2\angle 60^\circ}{2} = \boxed{1\angle 60^\circ}$

(f)



At node a,

$$\frac{V_a}{1K} + \frac{V_a - V_o}{5K} = 0$$

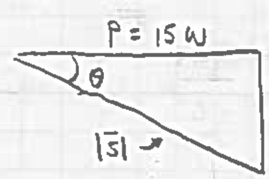
and $V_a = 3v$

$$\frac{3}{1K} + \frac{3 - V_o}{5K} = 0$$
$$15 + 3 - V_o = 0$$
$$V_o = 18$$

However, the op-amp will be saturated because the power supply voltages are only ± 12 v.

Therefore, $\boxed{V_o = 12 \text{ v.}}$

(g) A leading PF tells us that $\theta_I > \theta_V$, so the power angle $\theta = \theta_V - \theta_I$ is negative



Power triangle

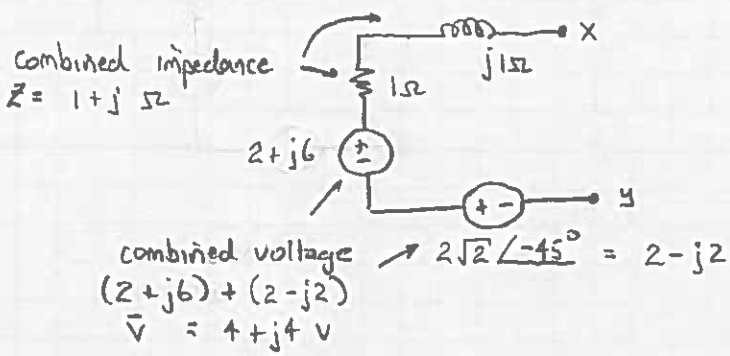
With a leading PF of 0.7, $\theta = -\cos^{-1}(0.7)$
$= -45.57°$

From the triangle, $Q = P \cdot \tan(\theta)$
$= 15 \times \tan(-45.57°)$
$= -15.30$ VAR

and apparent power $|\bar{S}| = \sqrt{P^2 + Q^2}$

$|\bar{S}| = 21.43$ VA

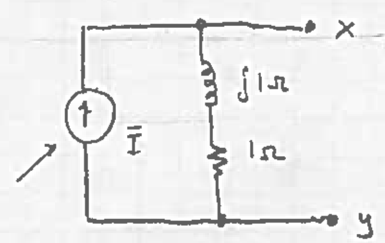
(h) Rearranging the series sources and impedances



Source transformation

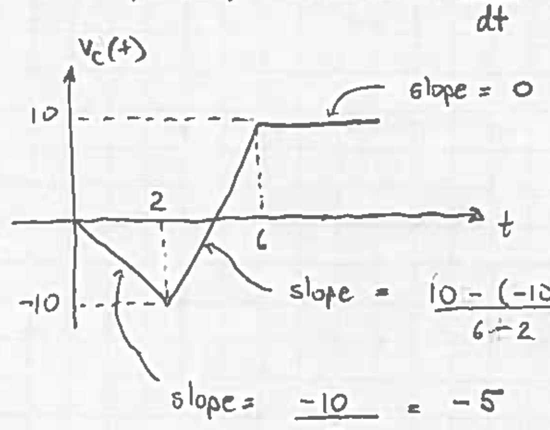
$\frac{\bar{V}}{\bar{Z}} = \frac{4+j4}{1+j1}$

$\bar{I} = 4$ A

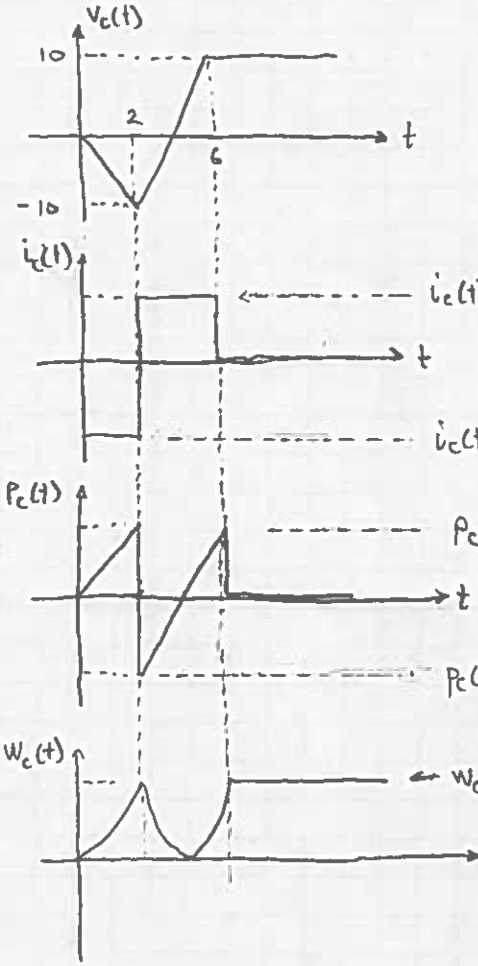


<u>Question 2</u>

(a) For the capacitor, $i(t) = C\frac{dv(t)}{dt}$, where $C = 0.1$ F



The current, power, and energy waveforms



$i_c(t) = 0.1 \dfrac{dv_c(t)}{dt} = 0.1 \times 5 = 0.5\,A$

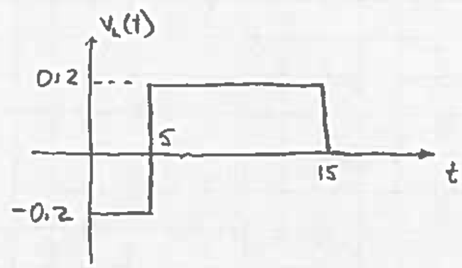
$i_c(t) = -0.5\,A$

$p_c(t) = v_c(t)\, i_c(t) = 10 \times 0.5 = 5\,W$

$p_c(t) = -5\,W$

$w_c(t) = \frac{1}{2} C v_c^2(t) = \frac{1}{2}(0.1)(10^2) = 5\,J$

(b) For the inductor, $i_L(t) = \dfrac{1}{L}\displaystyle\int_{t_1}^{t_2} v_L(t)\,dt + v_L(t_1)$, $L = 0.5\,H$



First interval: $0 \le t \le 5$, $i_L(t) = \dfrac{1}{0.5}\displaystyle\int_0^t (-0.2)\,dt + -1$ ← $i_L(0)$

$= 2\left[-0.2t\right]\Big|_0^t - 1$

$= -0.4t - 1$

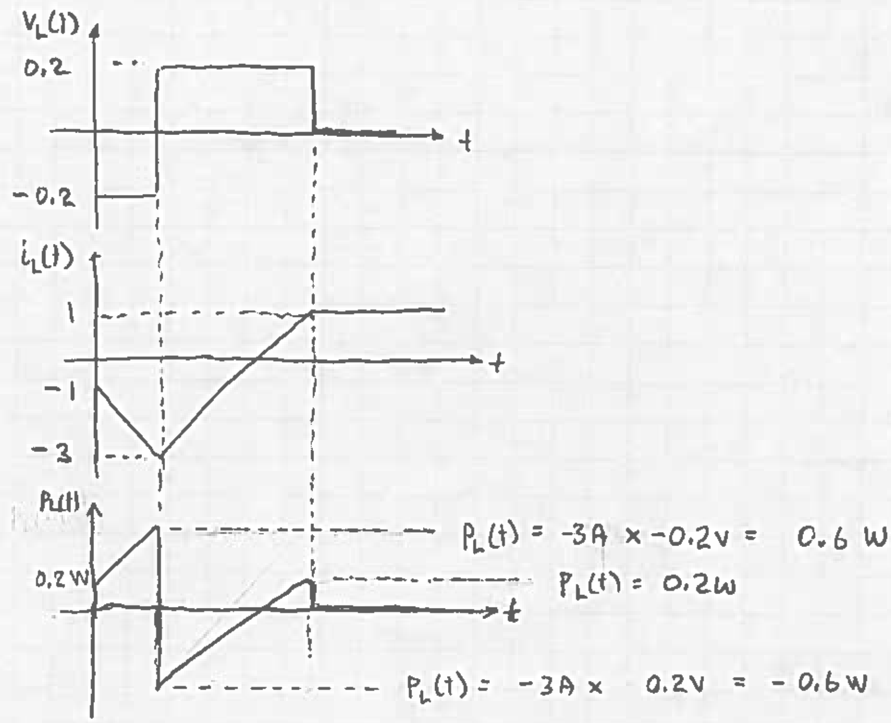
Second interval: $5 \leq t \leq 15$, $\quad i_L(t) = 2\int_5^t 0.2\,dt + i_L(5)$

where $i_L(5) = -0.4(5) - 1 = -3\text{ A}$

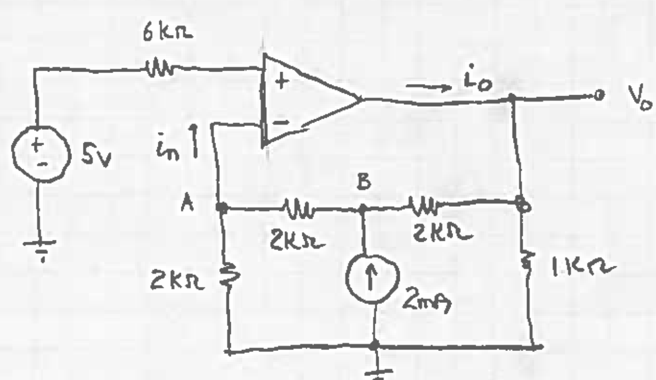
so $i_L(t) = 2\left[0.2t\right]_5^t - 3 = 0.4t - 0.4(5) - 3$
$= 0.4t - 5$

Finally, for $t \geq 15$, $\quad i_L(t) = i_L(15) = 0.4(15) - 5$
$= 1\text{ A}$

The current and power waveforms



<u>Question 3</u>



(a) Setting up for the node-voltage method

node A:
$$\frac{V_A}{2K} + \frac{V_A - V_B}{2K} + \cancel{i_n} = 0$$
$$2V_A - V_B = 0$$

so $V_B = 2V_A$, and we know $V_A = 5V$

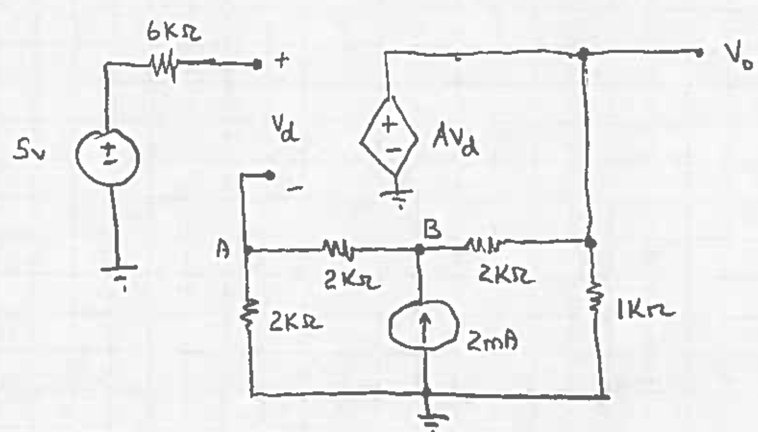
Then $V_B = 10$ V.

node B:
$$\frac{V_B - V_A}{2K} - 0.002 + \frac{V_B - V_O}{2K} = 0$$
$$V_B - V_A - 4 + V_B - V_O = 0$$

so
$$V_O = 2V_B - V_A - 4$$
$$= 2(10) - 5 - 4 = \boxed{11\text{ V}}$$

(b) At node V_o,
$$\frac{V_O}{1K} + \frac{V_O - V_B}{2K} - i_o = 0$$
$$2V_o + V_O - V_B = (2K) i_o$$
$$i_o = \frac{3V_O - V_B}{2K} = \frac{33 - 10}{2K} = \boxed{11.5\text{ mA}}$$

(c)



node A:
$$\frac{V_A}{2K} + \frac{V_A - V_B}{2K} = 0$$
$$2V_A - V_B = 0$$

so $V_B = 2V_A$, only this time, we don't know V_A. (1)

node B : unchanged from above $V_o = 2V_B - V_A - 4$

and from (1) $V_o = 2(2V_A) - V_A - 4$

$= 3V_A - 4$ (2)

At the op-amp input terminals, $V_d = 5 - V_A$

and $V_o = A V_d = 100(5 - V_A)$ (3)

so $V_o = 500 - 100 V_A$

$$V_A = \frac{V_o - 500}{-100} = \frac{500 - V_o}{100} \quad (4)$$

Substitute (4) into (3)

$$V_o = 3V_A - 4 = 3\left(\frac{500 - V_o}{100}\right) - 4$$

$$V_o = 15 - 0.03 - 4$$

$$V_o(1.03) = 11$$

so $\boxed{V_o = 10.68\ \text{v}}$ ← for an ideal opamp, this should be 11 v

and $V_d = 5 - V_A$, where, from (2),

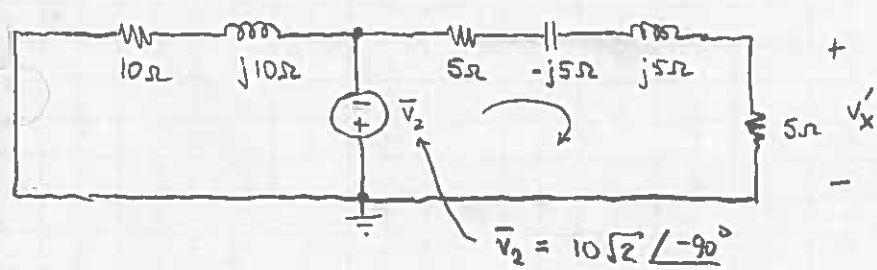
$V_o = 3V_A - 4$, so $V_A = \frac{V_o + 4}{3}$

$= \frac{10.68 + 4}{3} = 4.89$ v

Finally $V_d = 5 - 4.89 = \boxed{0.11\ \text{v}}$ ← for an ideal opamp, this should be zero.

Question 4 [revised solution]

(a) Let's zero $\bar{V}_1$ and $\bar{I}_1$ to start

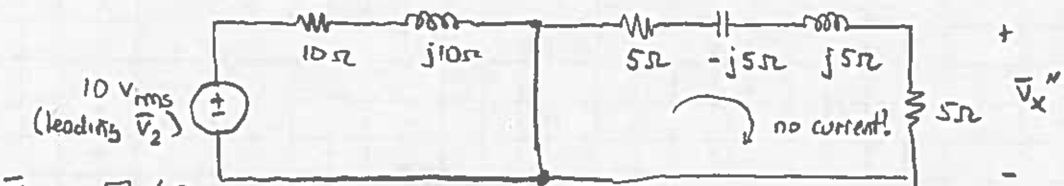


In the right-hand portion of this circuit, we have a simple voltage divider

$$\bar{V}_x' = -\bar{V}_2 \times \frac{5}{5 - j5 + j5 + 5} = 0.5\bar{V}_2$$

$$\bar{V}_2 = 10\sqrt{2}\angle{-90^\circ}, \quad \text{so} \quad \bar{V}_x' = -5\sqrt{2}\angle{-90^\circ} = 5\sqrt{2}\angle{90^\circ}$$

Next, let's zero $\bar{V}_2$ and $\bar{I}_1$

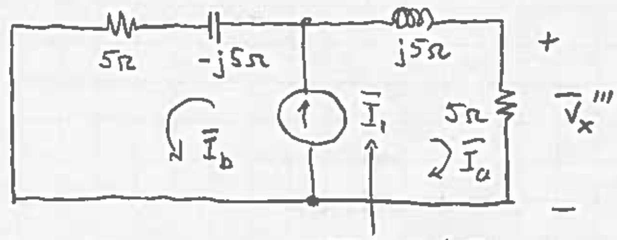


$\bar{V}_1 = 10\sqrt{2}\angle{0^\circ}$

There is no voltage or current in the right-hand portion of this circuit!

$$\bar{V}_x'' = 0.$$

Finally, the current source $\bar{I}_1$ acting alone



$\bar{I}_1 = 2\angle{0^\circ}$

Here, we have a current divider, and we need $\bar{I}_a$ to find

$$\bar{V}_x''' = 5\,\bar{I}_a$$

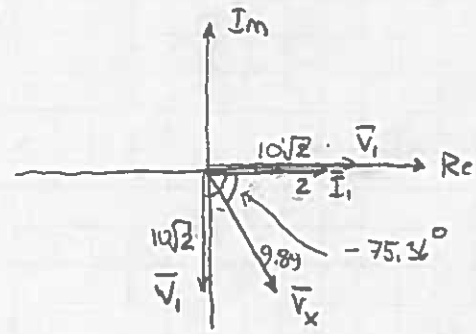
By inspection, $$\bar{I}_a = \frac{(5 - j5) \times 2}{5 - j5 + j5 + 5} = \frac{10 - j10}{10}$$

giving $$\bar{V}_x''' = \frac{5\,(10 - j10)}{10} = 5 - j5 = 5\sqrt{2}\angle{-45^\circ}$$

By superposition, $\bar{V}_x = \bar{V}_x' + \bar{V}_x'' + \bar{V}_x'''$

$$\bar{V}_x = 5\sqrt{2}\angle 90^\circ + 0 + 5 - j5$$
$$= j5\sqrt{2} + 5 - j5$$
$$= 5 + j\,2.071 = 5.42\angle 22.5^\circ$$

(b) Phasor diagram



(c) $\bar{V}_2 = 10\sqrt{2}\angle -90^\circ$, and $\bar{I} = 5\sqrt{2}\angle -45^\circ$

Complex power $\bar{S} = \frac{1}{2}\bar{V}_2\bar{I}^* = \frac{1}{2}(10\sqrt{2}\angle -90^\circ)(5\sqrt{2}\angle 45^\circ)$

$$\bar{S} = 50\angle -45^\circ$$

or in rectangular representation

$$\bar{S} = P + jQ = 50\cos(-45^\circ) + 50\sin(-45^\circ)$$

$$P = 50/\sqrt{2} \text{ W}$$
$$Q = -50/\sqrt{2} \text{ VAR}$$

The power angle $\theta = -45^\circ$, so power factor is $1/\sqrt{2}$ leading

Question 5

(a) Under full load, the motor is providing $P_{out} = 15$ HP at 1200 rpm

The torque $T_{out} = \dfrac{P_{out}}{\omega_m} = \dfrac{15 \times 746}{1200 \times 2\pi/60} = 89.047$ Nm

Since there are no rotational losses, $P_{dev} = P_{out}$

Losses in the armature and field

$$P_F = \frac{V_T^2}{R_F} = \frac{250^2}{50} = \boxed{1250\ W}$$

$$I_A = I_L - I_F = 54.5 - \frac{250}{50} = 49.5\ A$$

$$P_A = I_A^2 R_A = (49.5)^2 \times 0.484 = \boxed{1185.9\ W}$$

The induced voltage, by KVL,

$$E_A = V_T - I_A R_A = 250 - (49.5)(0.484)$$

$$= \boxed{226.04\ V}$$

Efficiency:

$$P_{in} = V_T I_L = (250)(54.5) = 13{,}625\ W$$

$$P_{out} = 15\ HP \times 746 = 11{,}190\ W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \boxed{82.1\%}$$

(b) With $n_m = 0$, $E_A = 0$ since $E_A = K\phi\omega_m$.

From part (a), we know the machine constant

At $n_m = 1200$ rpm, $E_A = 226.04$ V

$$K\phi = \frac{E_A}{\omega_m} = \frac{226.04}{1200 \times 2\pi/60} = 1.799$$

Because $E_A = 0$, $I_A = \frac{V_T - E_A}{R_A} = \frac{250}{0.484} = 516.53\ A$

so $T_{dev} = K\phi I_A = 1.799 \times 516.53 = 929.12\ Nm$

$T_{out} = T_{dev} = \boxed{929.12\ Nm}$ $\boxed{P_{out} = 0\ W}$

since $n_m = 0$.
$\omega_m = 0$.

(c) Assuming the same T_{out}, and adding T_{rot} to it:

$$T_{dev} = T_{out} + T_{rot} = 89.047 + 10 = 99.047 \text{ Nm}$$

This will cause an increase in armature current

$$I_A = \frac{T_{dev}}{K\phi} = \frac{99.047}{1.799} = 55.057 \text{ A}$$

And by KVL,
$$E_A = V_T - I_A R_A = 250 - (55.057)(0.484)$$
$$= 223.35 \text{ v}$$

And
$$\omega_m = \frac{E_A}{K\phi} = \frac{223.35}{1.799} = 124.15 \text{ rad/se}$$

$$n_m = \omega_m \times \frac{60}{2\pi} = \boxed{1185.58 \text{ rpm}}$$