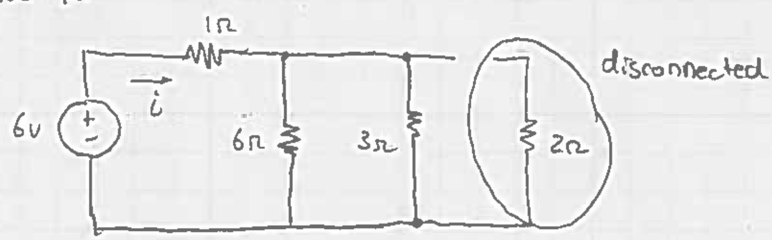
Question 1

(a) Since the source is DC, the inductor will behave as a short circuit, and the capacitor as open circuit. The circuit simplifies to



Total equivalent resistance : $R_{eq} = 3 \parallel 6 + 1$
$= 2 + 1 = 3\Omega$

Therefore $i = \dfrac{6\,v}{3\,\Omega} = \boxed{2\,A}$

(b) For circuit element x,

- The phasor diagram reveals that current $\bar{I}$ leads voltage $\bar{V}_1$ by 90°
- For a capacitor, $\bar{I} = \dfrac{\bar{V}_1}{Z_c \angle -90^\circ} = \bar{V}_1 \times \dfrac{1}{Z_c} \angle 90^\circ$

so $\bar{I}$ leads $\bar{V}_1$ by 90°. x is a $\boxed{\text{capacitor}}$

For circuit element y,

- The phasor diagram reveals that current $\bar{I}_y$ and voltage $\bar{V}_2$ are in phase, implying that:

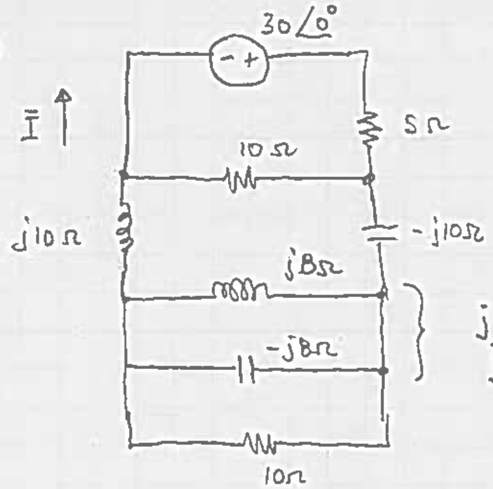
y is a $\boxed{\text{resistor}}$. $\bar{V}_2 = R \cdot \bar{I}_y$
real

For circuit element z,

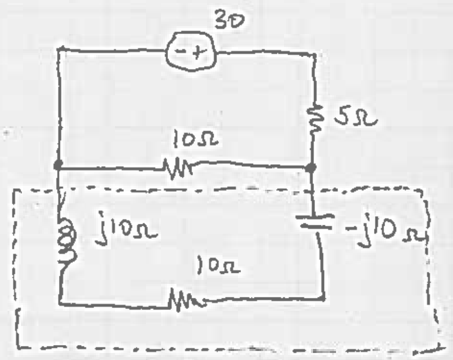
- The phasor diagram reveals that the current $\bar{I}_z$ lags the voltage $\bar{V}_2$ by 90°
- For an inductor, $\bar{I}_z = \dfrac{\bar{V}_2}{Z_L \angle 90^\circ} = \bar{V}_2 \times \dfrac{1}{Z_L} \angle -90^\circ$

so $\bar{I}_z$ lags $\bar{V}_2$ by 90°. z is an $\boxed{\text{inductor}}$

(c)



$$\frac{j8 \times -j8}{j8 - j8} = \infty \text{ (open circuit)}$$



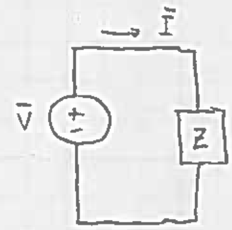
Series: $j10 - j10 + 10 = 10\Omega$

This leaves the total equivalent impedance across the source Z_{eq}

$$Z_{eq} = 10 \,//\, 10 + 5$$
$$= 10\Omega$$

Therefore, $\bar{I} = \dfrac{30\angle 0°}{10} = \boxed{3\angle 0° \text{ A}}$

(d)



We have $\bar{S} = \frac{1}{2}\bar{V}\bar{I}^* = 12\angle 70°$

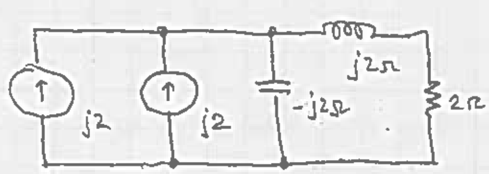
with $\bar{V} = 6\angle 40°$, $\quad \bar{I}^* = \dfrac{\bar{S}}{\frac{1}{2}\bar{V}}$

$$\bar{I}^* = \frac{12\angle 70°}{\frac{1}{2} \times 6\angle 40°} = 4\angle 30°$$

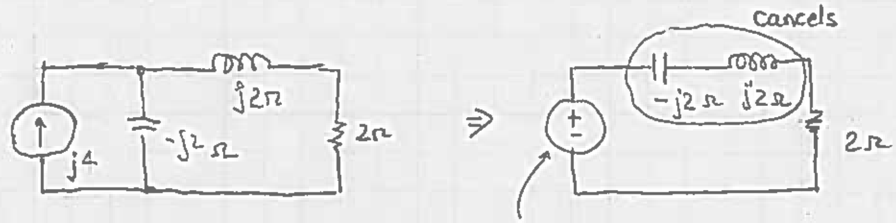
Therefore, $\boxed{\bar{I} = 4\angle -30°}$ and $Z = \dfrac{\bar{V}}{\bar{I}} = \dfrac{6\angle 40°}{4\angle -30°}$

$$\boxed{1.5\angle 70°}$$

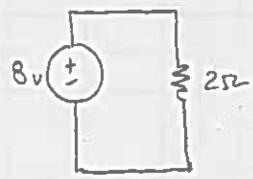
(e) Rearranging the circuit gives



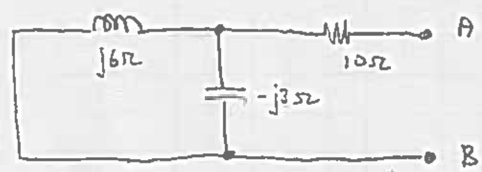
Combining the two current sources ...



$j4 \times -j2 = 8V$



(f) Since there are no dependent sources, we may zero the independent ones



We have $Z_t = j6 \,//\, -j3 + 10 = \dfrac{j6 \times -j3}{j6 - j3} + 10$

$= \dfrac{18}{j3} + 10$

$Z_t = 10 - j6$

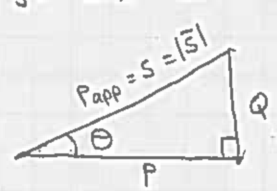
Therefore x (or y) is a $\boxed{10\Omega \text{ resistor}}$

y (or x) is ... $-j6 = \dfrac{-j}{\omega C}$

so $C = \dfrac{1}{6\omega} = \dfrac{1}{6 \times 2} = \dfrac{1}{12} = 0.0833\ F$

$\boxed{0.0833\ F \text{ capacitor}}$

(g) A lagging PF implies a positive power angle (inductive). The power triangle will look like



We know that $P = 7460\ W$ and that $\cos(\theta) = 0.7$

Therefore $\boxed{\theta = +45.57^\circ}$

And $\frac{Q}{P} = \tan(\theta)$, so $Q = P\tan(45.57^\circ)$

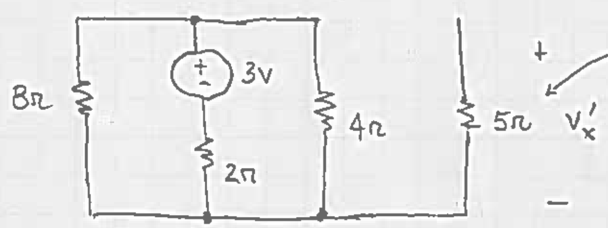
$$\boxed{Q = 7610.7\ VAR}$$

Complex power $\bar{S} = P + jQ = \boxed{\begin{array}{l} 7460 + j\,7610.7 \\ = 10657\angle 45.57^\circ\ VA \end{array}}$

Apparent power $P_{app} = |\bar{S}| = \sqrt{P^2 + Q^2} = S$

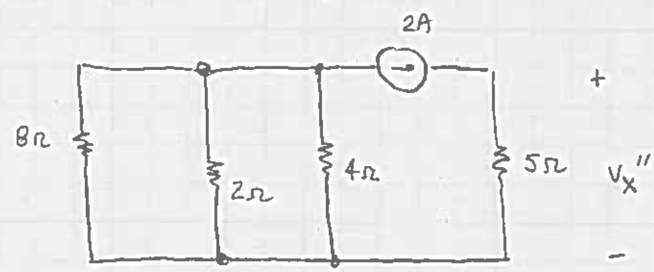
$$P_{app} = \boxed{10657\ VA}$$

(h) Consider the voltage source by itself



$V_x' = 0$

Now the current source by itself



The current in the 5Ω resistor is simply 2A, irrespective of the rest of the circuit.

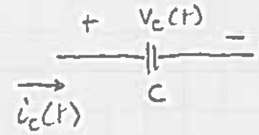
$$V_x'' = 2A \times 5\Omega = 10V$$

By superposition, $V_x = V_x' + V_x''$

$$\boxed{V_x = 10\ V}$$

Question 2

(a)



$C = \frac{1}{3}$ F

During the time interval $0 \le t < 6$ secs,

$$v_c(t) = \frac{1}{C}\int_0^t i_c(x)\,dx + v_c(0)$$

During this interval, $i_c(t) = \left(\frac{0.4}{6}\right)t = 0.0667t$

$$\text{so} \quad v_c(t) = 3\int_0^t 0.0667x\,dx + 1.4$$

$$= 0.1x^2\Big|_0^t + 1.4$$

$$v_c(t) = 0.1t^2 + 1.4$$

Thus, the voltage is 1.4 V when $t = 0$, and reaches a peak of

$$v_c(6) = 0.1(6^2) + 1.4 = 5\text{v}$$

The next interval is $6 \le t < 12$, where

$$i_c(t) = -0.5 \text{ A}$$

Then,

$$v_c(t) = 3\int_6^t (-0.5)\,dx + v_c(6)$$

$$= -1.5x\Big|_6^t + 5$$

$$= -1.5(t-6) + 5$$

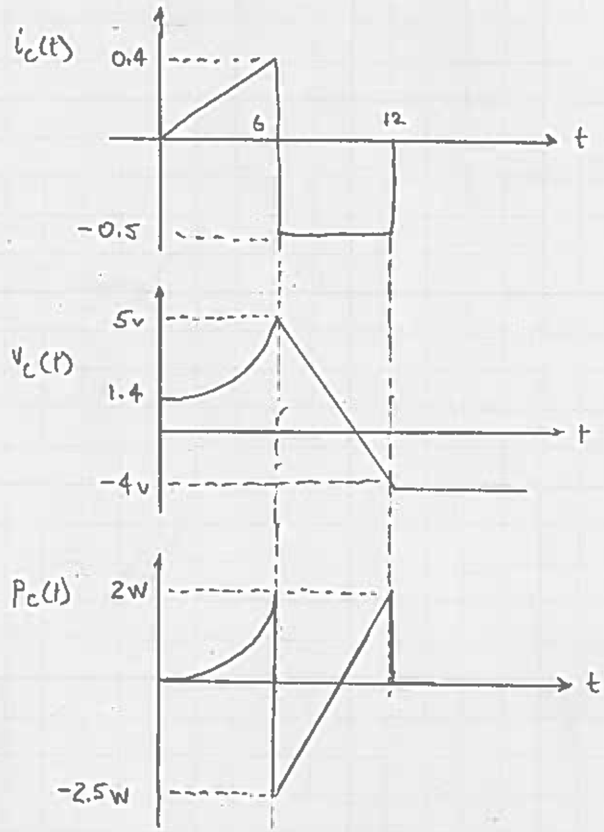
$$= -1.5t + 9 + 5$$

$$= -1.5t + 14$$

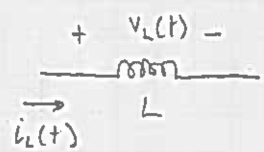
This evaluates to $v_c(6) = -9 + 14 = 5$ and $v_c(12) = -18 + 14 = -4$v.

Finally, for $t \ge 12$, $i_c(t) = 0$, and the capacitor maintains a constant voltage of $v_c(12) = -4$v

Sketching the resulting voltage and power waveforms ...



(b)



$L = 0.2\,H$

During the interval $0 \le t < 4$, $i_L(t)$ has a slope of $20/4 = 5$.

Therefore $v_L(t) = L\dfrac{di_L(t)}{dt} = 0.2 \times 5 = 1\,v$

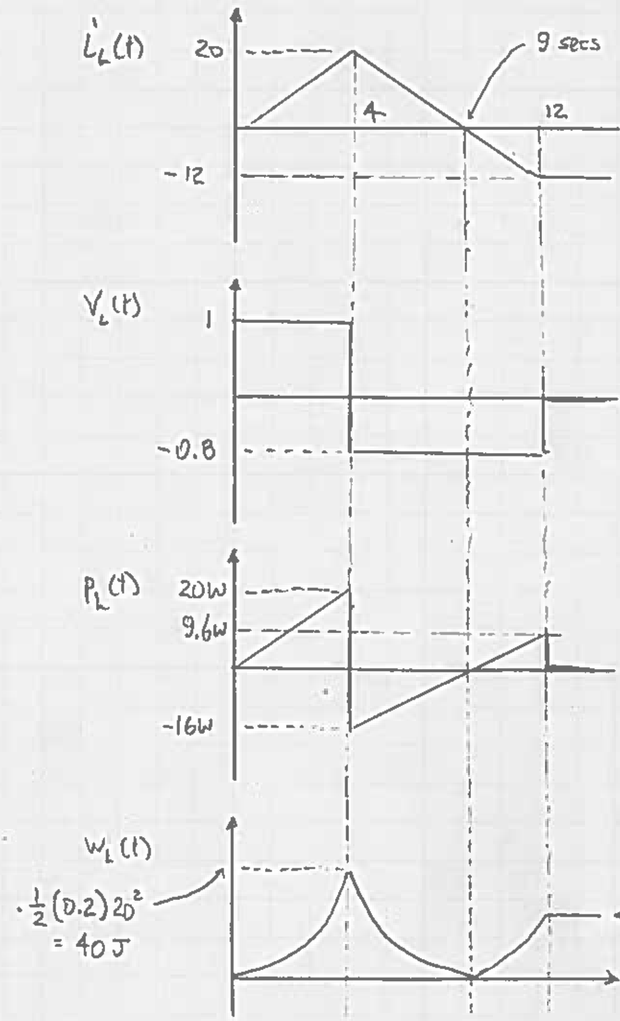
During the next interval $4 \le t < 12$, $i_L(t)$ has a slope m

$$m = \frac{-12-20}{12-4} = -4$$

so that $v_L(t) = 0.2 \times -4 = -0.8\,v$

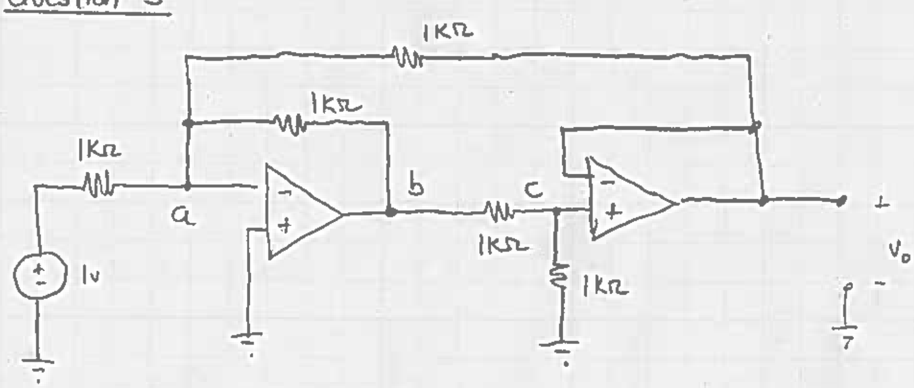
Finally, for $t \ge 12$, slope $= 0$ and $v_L(t) = 0$

Sketching the resulting voltage, power, and energy waveforms ..



$w_L(t) = \frac{1}{2} L\, i^2(t)$

Question 3



The node-voltage method gives:

Node a : $\frac{V_a - 1}{1000} + \frac{V_a - V_b}{1000} + \frac{V_a - V_o}{1000} = 0$

We know $V_a = 0$, so

$$0 - 1 \;+\; 0 - V_b \;+\; 0 - V_o = 0$$

$$V_o + V_b = -1 \qquad (1)$$

Node c: $$\frac{V_c - V_b}{1000} + \frac{V_c}{1000} = 0$$

$$2V_c = V_b \qquad (2)$$

We also know that $V_c = V_o$ from the second op-amp, so equation (2) becomes

$$V_b = 2V_o$$

Substitute into (1) $\quad V_o + 2V_o = -1$

$$\boxed{V_o = -\frac{1}{3}\ \text{V}}$$

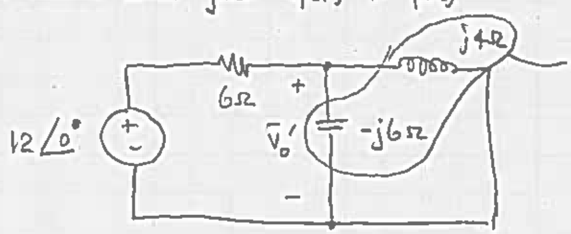
Question 4

Impedances in the circuit for $\omega = 2$ rads/sec

inductor: $Z_L = j\omega L = j \times 2 \times 2\text{H} = j4\Omega$

capacitor: $Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{j \times 2 \times \frac{1}{12}} = -j6\Omega$

The circuit with just $v_1(t)$ is play



parallel $\dfrac{j4 \times -j6}{j4 - j6}$

$= \dfrac{24}{-j2} = j12\Omega$

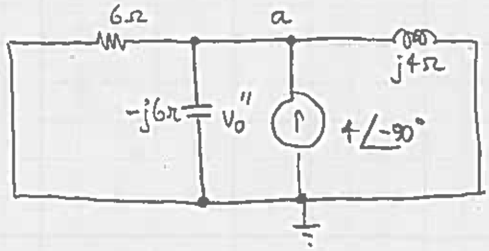
Then, by voltage division $\quad \bar{V}_o' = \dfrac{j12}{6 + j12} \times 12$

$$\bar{V}_o' = \frac{j2}{1 + j2} \times \frac{1 - j2}{1 - j2} \times 12$$

$$\bar{V}_o' = \frac{j2 + 4}{1 - j2 + j2 + 4} \times 12 = \frac{48 + j24}{5} = 9.6 + j4.8$$

or $10.73 \angle 26.56^\circ$

The circuit with just $i_1(t)$ energized



Node-voltage method: Node a: $\frac{\bar{V}_a}{6} + \frac{\bar{V}_a}{-j6} + \frac{\bar{V}_a}{j4} - 4\angle{-90^\circ} = 0$

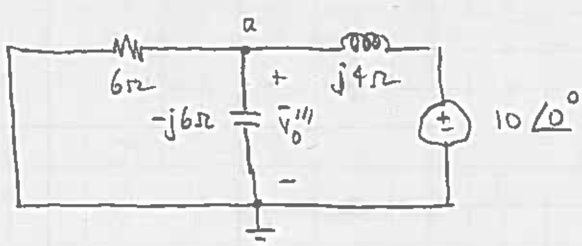
$$(\times 24) \quad \bar{V}_a(4 + j4 - j6) = 96\angle{-90^\circ}$$
$$\bar{V}_a(4 - j2) = -j96$$

$$\bar{V}_a = \frac{-j96}{4-j2} = \frac{-j48}{2-j} \times \frac{2+j}{2+j}$$

$$\bar{V}_a = \frac{-j96 + 48}{4 - 2j + 2j + 1} = \frac{-j96 + 48}{5}$$

$$\text{so} \quad \bar{V}_o'' = \bar{V}_a = 9.6 - j19.2 = 21.47\angle{-63.4^\circ}$$

Finally, with just $v_2(t)$



At node a: $\frac{\bar{V}_a}{6} + \frac{\bar{V}_a}{-j6} + \frac{\bar{V}_a - 10}{j4} = 0$

$$(24) \quad 4\bar{V}_a + j4\bar{V}_a - j6\bar{V}_a + j60 = 0$$
$$\bar{V}_a(4 + j4 - j6) = -j60$$
$$\bar{V}_a(4 - j2) = -j60$$

$$\text{so} \quad \bar{V}_a = \frac{-j60}{4-j2} = \frac{-j30}{2-j} \times \frac{2+j}{2+j} = \frac{-j60 + 30}{4 + j2 - j2 + 1}$$

$$V_o''' = \bar{V}_a = \frac{-j60 + 30}{5} = 6 - j12 = 13.42\angle{-63.43^\circ}$$

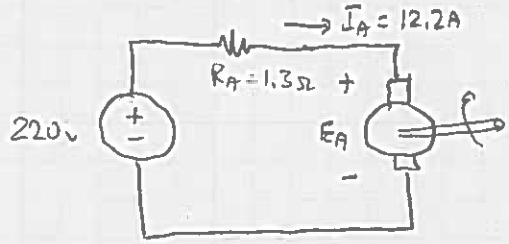
Finally,

$$V_o = V_o' + V_o'' + V_o''' = (9.6 + j4.8) + (9.6 - j19.2) + (6 - j12)$$

$$= 25.2 - j26.4 = \boxed{36.50\angle{-46.33^\circ}}$$

Question 5

(a) In the armature circuit, at full load.



$P_{out} = 3HP$
$= 3 \times 746 = 2238$ W

$n_m = 950$ rpm,
so $\omega_m = \frac{2\pi(950)}{60} = 99.484$ r/s

By KVL, $E_A = 220 - I_A R_A$
$= 220 - (12.2)(1.3) = 204.14$ V

$P_{dev} = I_A E_A = (12.2)(204.14) = \boxed{2490.508 \text{ W}}$

And $T_{dev} = P_{dev} / \omega_m = 2490.508 / 99.484 = \boxed{25.034 \text{ Nm}}$

The armature power loss is $P_A = I_A^2 R_A$
$= (12.2)^2 (1.3) = \boxed{193.492 \text{ W}}$

Rotational power losses will be $P_{rot} = P_{dev} - P_{out}$
$= 2490.508 - 2238$

$P_{rot} = \boxed{252.508 \text{ W}}$

(b) Removing the mechanical load leaves only the rotational losses as the developed torque.

$T_{rot} = P_{rot} / \omega_m = 252.508 / 99.484$
$= 2.538$ Nm

From part (a), we may determine the machine constant

$K\phi = \frac{E_A}{\omega_m} = \frac{204.14}{99.484} = 2.052$

The new armature current is $I_A = \frac{T_{dev}}{K\phi} = \frac{T_{rot}}{K\phi}$

$I_A = \frac{2.538}{2.052} = 1.237$ A

And $E_A = 220 - I_A R_A = 220 - (1.237)(1.3)$
$= 218.392\ V$

Therefore, $\omega_m = \frac{E_A}{K\phi} = \frac{218.392}{2.052} = 106.43\ r/s$

so $n_m = 60 \times \omega_m / 2\pi = \boxed{1016.3\ rpm}$

(c) Halving I_F causes the machine constant to halve

New $K\phi = 1.026$

Neither the output torque nor rotational torque loss change, so the total developed torque is unchanged from part (a)

$T_{dev} = 25.034\ Nm$

The new armature current $I_A = \frac{T_{dev}}{K\phi} = \frac{25.034}{1.026} = 24.4$

Giving a new $E_A = 220 - I_A R_A = 188.28\ V$

and $\omega_m = \frac{E_A}{K\phi} = \frac{188.28}{1.026} = 183.51\ r/s$

From part (a) $T_{out} = P_{out} / \omega_m = 2238 / 99.484$
$= 22.496\ Nm$

Finally, $P_{out} = T_{out}\,\omega_m = (22.496)(183.51)$
$= 4128.27\ W$

$P_{out} = \boxed{5.534\ HP}$