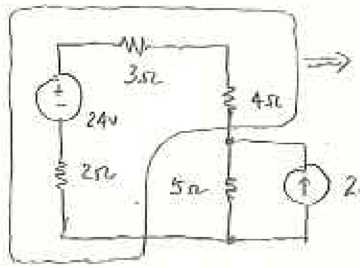
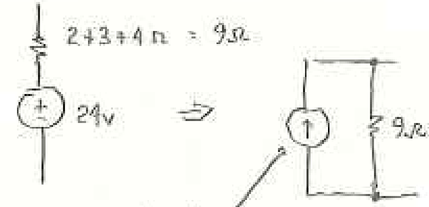
Question 1.

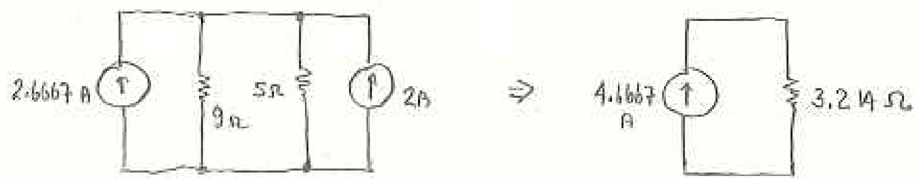
(a)



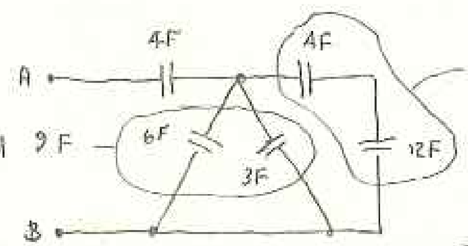


$2+3+4\,\Omega = 9\,\Omega$

$i' = \frac{24}{9} = 2.6667\,A$

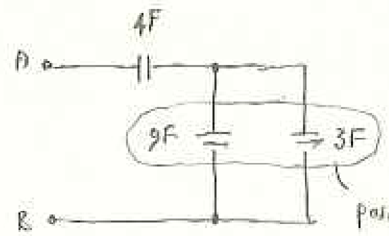


(b)



series $\frac{4 \times 12}{4+12} = 3F$

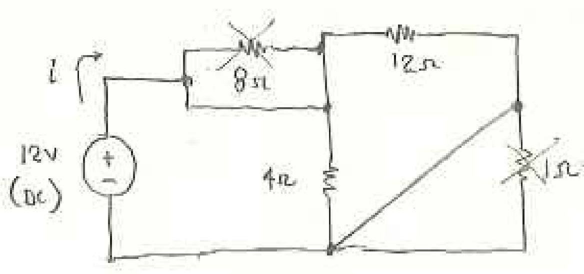
parallel 9 F

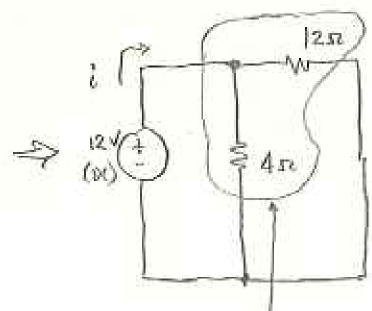


parallel 12 F

$$C_{eq} = \frac{4 \times 12}{4 + 12} = 3F$$

(c) For DC, capacitors look like open circuits and inductors look like a short circuit

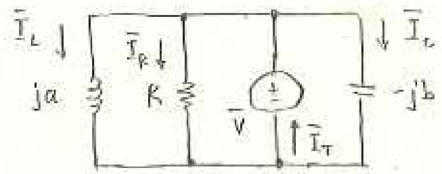




parallel $\frac{12 \times 4}{12+4} = 3\,\Omega$

So $i = \frac{12V}{3\,\Omega} = 4A$

(d)



Since $a = b$, the inductor and capacitor are in parallel resonance. We have

$$\bar{I}_L = \frac{\bar{V}}{ja}, \quad \bar{I}_C = \frac{\bar{V}}{-jb}, \quad \bar{I}_R = \frac{\bar{V}}{R}$$

$$\text{and} \quad \bar{I}_T = \bar{I}_L + \bar{I}_C + \bar{I}_R = \cancel{\frac{\bar{V}}{ja} - \frac{\bar{V}}{jb}} + \frac{\bar{V}}{R}$$

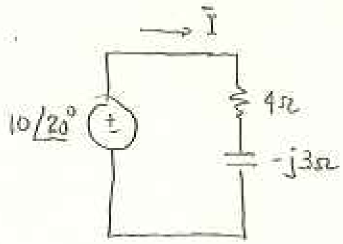
cancels.

$$\bar{I}_T = \bar{I}_R = \frac{\bar{V}}{R}$$

Since the inductor current cancels the capacitor current, the total current is simply the resistor current.

$\bar{I}_R$ and $\bar{I}_T$ are in phase, so $\theta = \theta_T - \theta_R = 0°$

(e)



We have $\bar{I} = \dfrac{10\angle 20°}{4 - j3}$

or $\bar{I} = \dfrac{10\angle 20°}{5\angle -36.87°}$

$= 2\angle 56.87°$

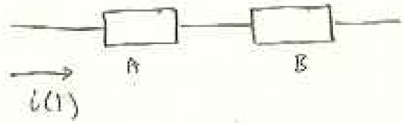
Complex power $\bar{S} = \frac{1}{2}\bar{V}\bar{I}^* = \frac{1}{2}(10\angle 20°)(2\angle -56.87°)$

$\bar{S} = 10\angle -36.87° = 8 - j6$ VA

$P = 8$ $\quad Q = -6$

The current leads the voltage, so this is a leading power factor.

(f) The impedance



We are given $v(t) = 10 \sin(100t + 30^\circ) = 10 \cos(100t + 30^\circ - 90^\circ)$

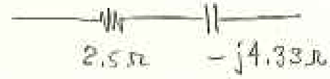
so $\bar{V} = 10 \angle -60^\circ$

and also $i(t) = 2 \cos(100t)$, so $\bar{I} = 2 \angle 0^\circ$

Therefore $Z = \dfrac{\bar{V}}{\bar{I}} = \dfrac{10 \angle -60^\circ}{2 \angle 0^\circ} = 5 \angle -60^\circ$

or $Z = 2.5 - j\,4.33\ \Omega$

Therefore,

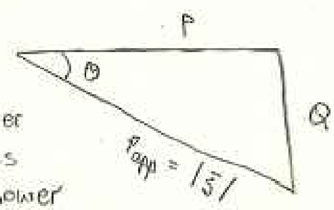


$-j4.33 = \dfrac{-j}{\omega C}$, $\omega = 100$ rad/s

so $C = \dfrac{1}{4.33\omega} = \dfrac{1}{4.33 \times 100}$

$C = 2.3094$ mF

(g) The power triangle



leading power factor implies a <u>negative</u> power angle.

The power factor is 0.6, so $\cos(\theta) = 0.6$

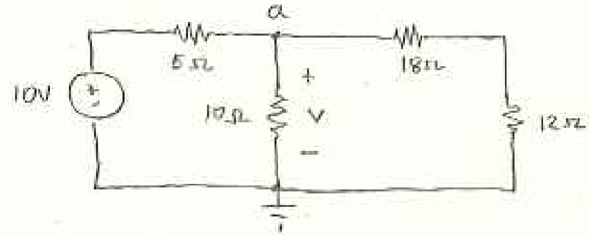
and $\theta = 53.13^\circ$

From the triangle $\cos(\theta) = \dfrac{P}{P_{app}}$, so $P_{app} = \dfrac{P}{\cos(\theta)}$

With $P = 6$ W, $P_{app} = \dfrac{6}{0.6} = 10$ VA

Question 2

For the voltage source acting alone,



The required voltage is at node a.

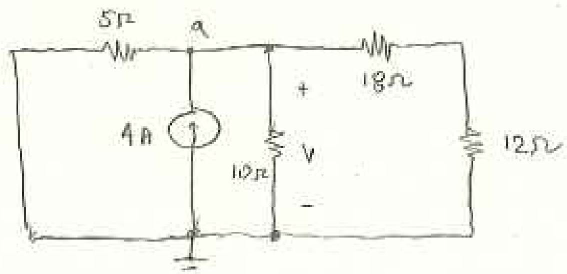
Node a: $\frac{V_a - 10}{5} + \frac{V_a}{10} + \frac{V_a}{18+12} = 0$

(×30) $6V_a - 60 + 3V_a + V_a = 0$

$10\,V_a = 60$

so $V_a' = 6\,V$.

For the current source alone,



Node a: $\frac{V_a}{5} + \frac{V_a}{10} - 4 + \frac{V_a}{30} = 0$

(×30) $6V_a + 3V_a + V_a = 120$

$10V_a = 120$

so $V_a'' = 12\,V$

By superposition, $V = V_a' + V_a''$

$V = 18\,V$.

Question 3

For a capacitor, $i = C\frac{dv}{dt}$, so we will be differentiating the voltage graph.

First interval: $0 \le t < 8$ secs

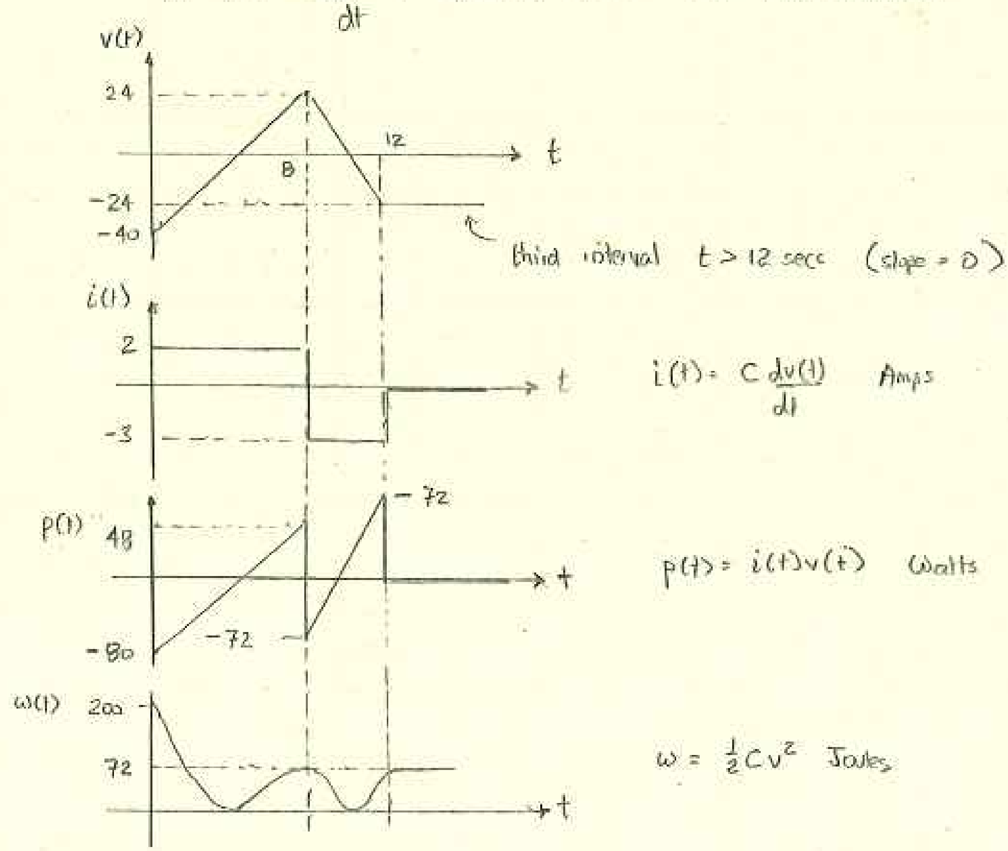
$$\text{slope} = \frac{24 - (-40)}{8} = 8$$

$$\text{so } i = C\frac{dv}{dt} = 0.25 \times 8 = 2A \text{ (Constant)}$$

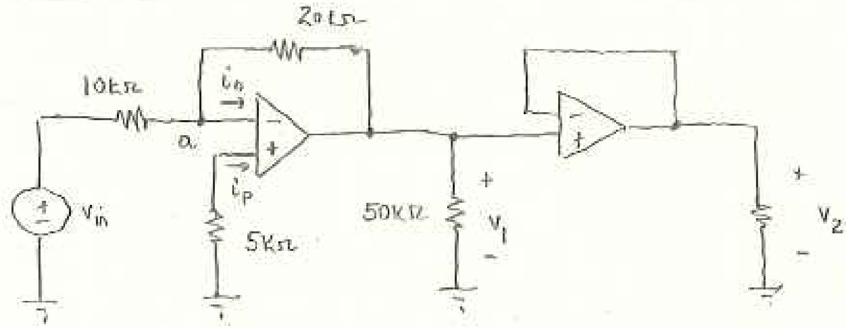
Second interval: $8 \le t < 12$ secs

$$\text{slope} = \frac{-24 - 24}{4} = -12$$

$$\text{so } i = C\frac{dv}{dt} = 0.25 \times -12 = -3A \text{ (also constant)}$$



Question 4



(a) In the first op-amp, at node a: $\frac{V_a - V_{in}}{10k} + \frac{V_a - V_1}{20k} + \cancel{i_n}^{0} = 0$

$$(\times 20k) \qquad 2V_a - 2V_{in} + V_a - V_1 = 0$$

But $V_a = 0$, so $-2V_{in} - V_1 = 0$

$$V_1 = -2V_{in}$$

The closed-loop gain $A_{v1} = \frac{V_1}{V_{in}} = -2$

In the second op-amp V_1 appears on both input terminals of the op-amp. Then, because the inverting input is connected directly to the output,

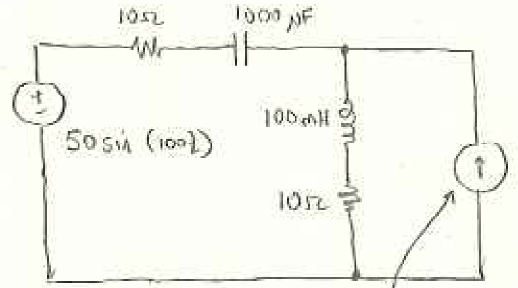
$$V_2 = V_1, \quad \text{so} \quad A_{v2} = \frac{V_2}{V_1} = 1$$

(b) Because the op-amps are powered by external voltage sources, the output V_1 and V_2 cannot exceed these voltages

Calculation	V_{in}	V_1	V_2	explanation
(i)	6.0	-12	0	first op-amp OK, second saturates
(ii)	-7.0	+14	+14	both op-amps OK
(iii)	8.0	-15	0	both op-amps saturate
(iv)	-9.0	+15	+15	first op-amp saturates, second OK

Question 5

(a)



$5/\sqrt{2}\cos(100t)$ rms Amps
$= \sqrt{2} \times 5/\sqrt{2}\cos(100t)$
$= 5\cos(100t)$

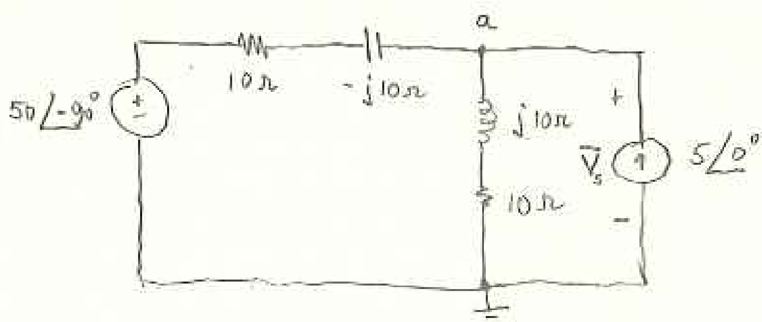
Impedances : 1000 μF capacitor $Z_C = \frac{1}{j\omega C} = \frac{1}{j(100)(1000\times10^{-6})}$

$= -j10\,\Omega$

100 mH inductor $Z_L = j\omega L = j(100)(0.1)$

$= j10\,\Omega$

Redrawing the circuit



At node a : $\frac{\bar{V}_a - 50\angle{-90^\circ}}{10-j10} + \frac{\bar{V}_a}{10+j10} - 5 = 0$

$[\times (10-j10)(10+j10)]$

$(\bar{V}_a - (-j50))(10+j10) + \bar{V}_a(10-j10) = 5(10-j10)(10+j10)$

$(\bar{V}_a + j50)(10+j10) + \bar{V}_a(10-j10) = 5(100+j100-j100+100)$

$10\bar{V}_a + \cancel{j10\bar{V}_a} + j500 - 500 + 10\bar{V}_a - \cancel{j10\bar{V}_a} = 1000$

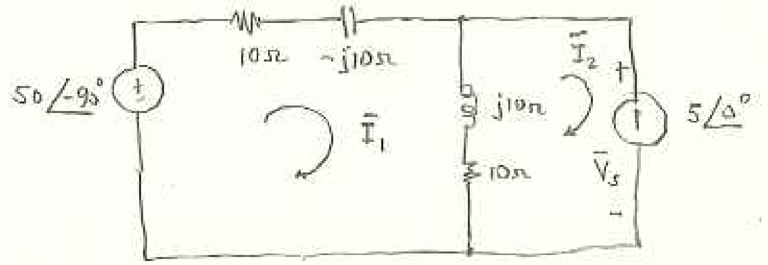
$20\bar{V}_a - 500 + j500 = 1000$

$$20\,\bar{V}_a = 1500 - j500$$

$$\bar{V}_a = 75 - j25 = 79.0569\angle{-18.435^\circ}$$

Therefore $\quad v_s(t) = v_a(t) = 79.0569\cos(100t - 18.435^\circ)$

Alternatively, using the mesh-current method



mesh $\bar{I}_2$: $\qquad \bar{I}_2 = -5\angle 0^\circ$

Mesh $\bar{I}_1$: $\qquad -50\angle{-90^\circ} + (10 - j10)\bar{I}_1 + (10 + j10)(\bar{I}_1 - \bar{I}_2) = 0$

$$j50 + 10\bar{I}_1 - \cancel{j10\bar{I}_1} + 10\bar{I}_1 + \cancel{j10\bar{I}_1} - 10\bar{I}_2 - j10\bar{I}_2 = 0$$

$$j50 + 20\bar{I}_1 - (10 + j10)\bar{I}_2 = 0$$

$$j50 + 20\bar{I}_1 - (10 + j10)(-5) = 0$$

$$20\bar{I}_1 = -50 - j50 - j50$$

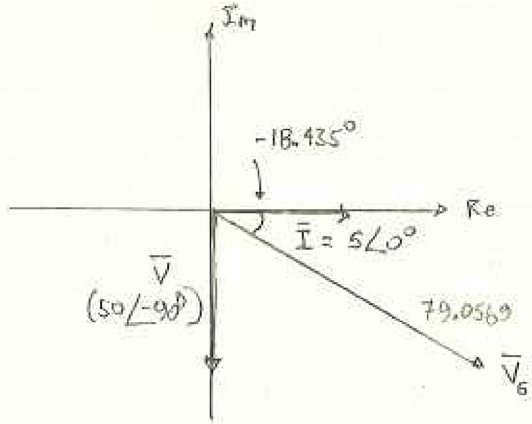
$$\bar{I}_1 = -2.5 - j5$$

And $\quad \bar{V}_s = (\bar{I}_1 - \bar{I}_2)(10 + j10) = (-2.5 - j5 + 5)(10 + j10)$

$$\bar{V}_s = (2.5 - j5)(10 + j10) = 25 - j50 + j25 + 50$$

$$\bar{V}_s = 75 - j25 = 79.0569\angle{-18.435^\circ}, \quad \text{as above}$$

(c) Phasor diagram



Question 6

(a) Output torque: $T_{out} = P_{out}/\omega_m$

where $\omega_m = 2\pi n_m/60 = 2\pi(1500)/60$
$= 157.0796$ rad/sec

so $T_{out} = \dfrac{10 \text{ HP} \times 746}{\omega_m} \approx 47.49$ Nm

Field power loss: $P_F = \dfrac{V_T^2}{R_F} = \dfrac{(300)^2}{150} = 600$ W

Armature power loss: $P_A = I_A^2 R_A$

where $I_A = I_L - I_F = 34.5 - \dfrac{V_T}{R_F} = 32.5$ A

giving $P_A = (32.5)^2(2.168\ \Omega) = 2289.95$ A

Induced voltage: $E_A = V_T - I_A R_A$
$= 300 - 32.5 \times 2.168 = 229.54$ V

Efficiency: $P_{out} = 10 \times 746 = 7460$ W
$P_{in} = V_T I_L = 300 \times 34.5 = 10350$ W

$\eta = \dfrac{7460}{10350} \times 100\% = 72.077\%$

(b) We may determine the machine constant from the information given

$k\phi = \dfrac{E_A}{\omega_m} = \dfrac{229.077}{157.0796} = 1.4613$

The increase in torque causes an increase in I_A

$I_A = \dfrac{T_{out}}{k\phi} = \dfrac{100}{1.4613} = 68.432$ A

Alternatively, we may determine I_A through proportionality

$\dfrac{T_1}{T_2} = \dfrac{I_{A1}}{I_{A2}} \rightarrow \dfrac{100}{47.49} = \dfrac{I_{A1}}{32.5}$

$I_{A1} = \dfrac{32.5}{47.49} \times 100 = 68.432$ A

This changes the induced voltage

$$E_A = V_T - I_A R_A$$
$$= 300 - (68.432)(2.168)$$
$$= 151.63 \text{ V}$$

which will change the speed

$$\omega_m = \frac{E_A}{K\phi} = \frac{151.63}{1.4613} \approx 103.765 \text{ rad/sec}$$

so $n_m = 60 \times \frac{\omega_m}{2\pi} = 990.88$ rpm