Question 1

(a) By quick inspection of the circuit,

$V_A = 12\text{ v}$
$V_C = 8\text{ v}$
$V_F = -6\text{ v}$

We'll need node-voltage equations (this is the easiest method)

Node B:
$$-2 + \frac{V_B - V_A}{4} + \frac{V_B - V_C}{2} = 0$$
$$-8 + V_B - V_A + 2V_B - 2V_C = 0$$

so
$$-8 + V_B - 12 + 2V_B - 16 = 0$$
$$3V_B = 36$$

$$\boxed{V_B = 12\text{ v}}$$

Node D:
$$-8 + \frac{V_D - V_C}{11} + \frac{V_D}{1} = 0$$
$$-88 + V_D - V_C + 11V_D = 0$$

so
$$-88 + V_D - 8 + 11V_D = 0$$
$$12V_D = 96$$

$$\boxed{V_D = 8\text{ v}}$$

Node E:
$$3 + \frac{V_E - V_F}{10} + \frac{V_E}{5} = 0$$
$$30 + V_E - V_F + 2V_E = 0$$

so
$$30 + V_E + 6 + 2V_E = 0$$
$$3V_E = -36$$

$$\boxed{V_E = -12\text{ v}}$$

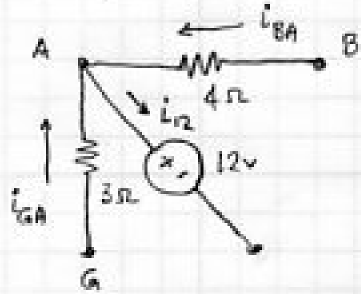
Node G:
$$4 + \frac{V_G - V_A}{3} + \frac{V_G - V_F}{6} = 0$$
$$24 + 2V_G - 2V_A + V_G - V_F = 0$$

so
$$24 + 2V_G - 24 + V_G + 6 = 0$$
$$3V_G = -6$$

$$\boxed{V_G = -2}$$

(b) Power in the sources

At A: $V_{12} = 12\text{V}$



$$i_{BA} = \frac{V_B - V_A}{4} = 0$$

$$i_{GA} = \frac{V_G - V_A}{3} = \frac{-2-12}{3} = -14/3 \text{ A}$$

Power $P_{12} = V_{12}\, i_{12} = 12 \times -\frac{14}{3} = -56\text{ W}$

At B: $P_{2A} = -2 \times 12 = -24\text{ W}$

At C: $i_{BC} = \frac{V_B - V_C}{2} = \frac{12-8}{2} = 2\text{ A}$

$i_{DC} = \frac{V_D - V_C}{11} = \frac{8-8}{11} = 0$

so $i_{8V} = i_{BC} + i_{DC} = 2\text{ A}$

$P_{8V} = 8 \times 2 = 16\text{ W}$

At D: $P_{8A} = -8 \times 8 = -64\text{ W}$

At E: $P_{3A} = 3 \times -12 = -36\text{ W}$

At F: $i_{GF} = \frac{V_G - V_F}{6} = \frac{-2+6}{6} = 2/3\text{ A}$

$i_{EF} = \frac{V_E - V_F}{10} = \frac{-12+6}{10} = -3/5\text{ A}$

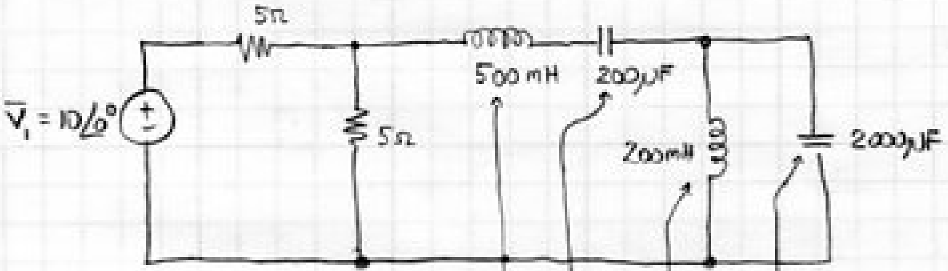
so $i_{6V} = \frac{2}{3} - \frac{3}{5} = \frac{1}{15}\text{ A}$

$P_{6V} = -6 \times \frac{1}{15} = -2/5\text{ W}$

At G: $P_{4A} = 4 \times -2 = -8\text{ W}$

Question 2

Consider $v_1(t)$ acting alone



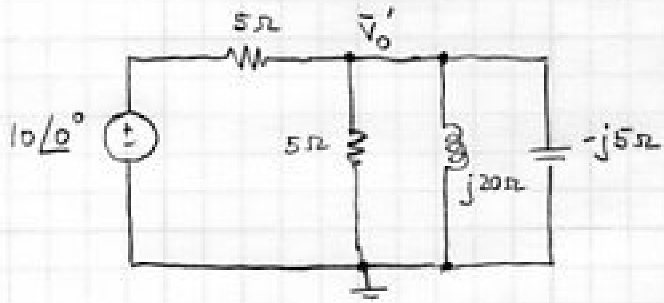
$$Z_L = j\omega L = j(100)\times 0.5 = j50\,\Omega$$

$$Z_C = 1/j\omega C = -j/(100\times 2000\times 10^{-6}) = -j5\,\Omega$$

$$Z_C = 1/j\omega C = -j/(100\times 200\times 10^{-6}) = -j50\,\Omega$$

$$Z_L = j\omega L = j(100)\times 0.2 = j20\,\Omega$$

Circuit simplifies to



At $\bar{V}_o'$,

$$\frac{\bar{V}_o' - 10}{5} + \frac{\bar{V}_o'}{j20} + \frac{\bar{V}_o'}{-j5} + \frac{\bar{V}_o'}{5} = 0$$

$$\frac{\bar{V}_o' - 10}{5} - j\frac{\bar{V}_o'}{20} + j\frac{\bar{V}_o}{5} + \frac{\bar{V}_o'}{5} = 0$$

$$4\bar{V}_o' - 40 - j\bar{V}_o' + 4j\bar{V}_o' + 4\bar{V}_o' = 0$$

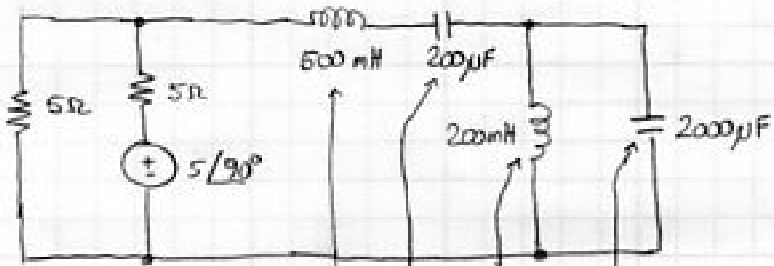
$$(8 + j3)\bar{V}_o' = 40$$

$$\bar{V}_o' = \frac{40}{8 + j3}$$

$$\text{so } \bar{V}_o' = \frac{40(8 - j3)}{8^2 + 3^2} = \frac{320 - j120}{73}$$

and $\bar{V}_o' = 4.3836 - j\,1.6438$
$= 4.6816 \angle -20.556^\circ$

Now consider $V_2(t)$ acting alone



$Z_L = j\omega L$
$= j(50)(0.5)$
$= j25\,\Omega$

$Z_C = 1/j\omega C$
$= -j/(50 \times 200 \times 10^{-6})$
$= -j100\,\Omega$

$Z_C = 1/j\omega C$
$= -j/(50 \times 2000 \times 10^{-6})$
$= -j10\,\Omega$

$Z_L = j\omega L$
$= j(50)(0.2)$
$= j10\,\Omega$

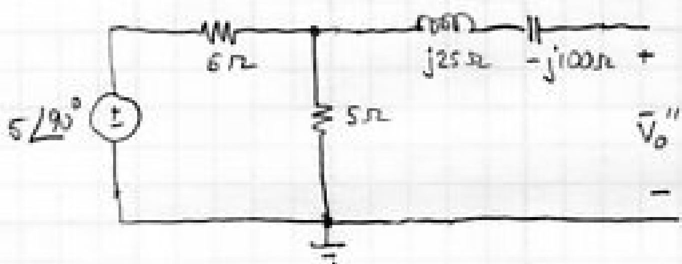
The parallel L/C connection gives

$$Z_{eq} = j10\,\Omega \,//\, -j10\,\Omega = \frac{(j10)(-j10)}{j10 - j10} = \infty$$

This is an open circuit!

Circuit simplifies to

no current flows!



by inspection; $\bar{V}_o'' = \dfrac{5}{5+5} \times 5\angle 90^\circ$

$= 2.5\angle 90^\circ$

Therefore, $v_o'(t) = 4.6816\cos(100t - 20.566°)$

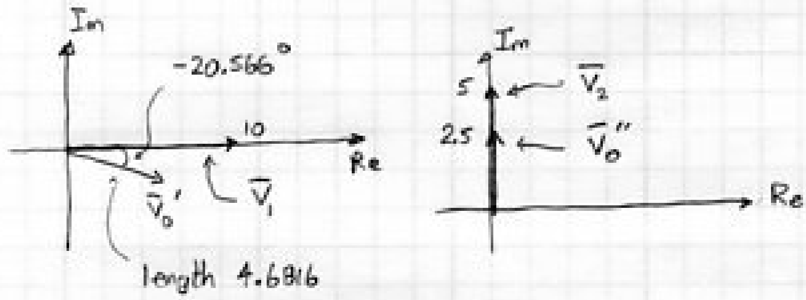
$v_o''(t) = 2.5\cos(50t + 90°)$

Finally, $v_o(t) = v_o'(t) + v_o''(t)$

$= 4.6816\cos(100t - 20.566°) + 2.5\cos(50t + 90°)$

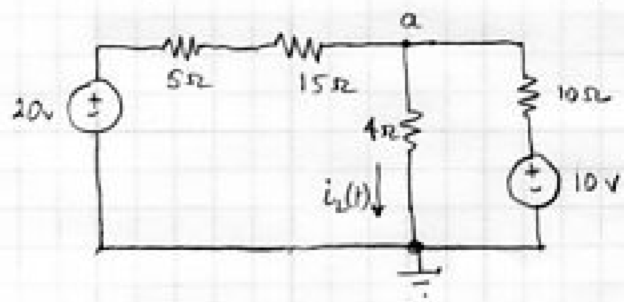
[cannot simplify more]

(b) Phasor diagrams



<u>Question 3</u>

(a) For $t<0$, the circuit in DC steady-state is



At node a, $\dfrac{V_a - 20}{5+15} + \dfrac{V_a}{4} + \dfrac{V_a - 10}{10} = 0$

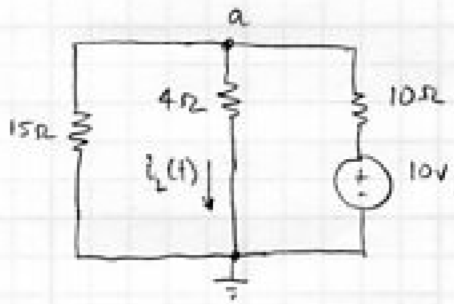
$V_a - 20 + 5V_a + 2V_a - 20 = 0$

$8V_a = 40$

$V_a = 5\text{ v}$

Therefore $I_0 = i_L(t)\big|_{t=0^-} = \dfrac{5\text{v}}{4\Omega} = 1.25\text{ A}$

Switch S_1 closes. At DC steady-state, $t \to \infty$,



At node a, $$\frac{V_a - 10}{10} + \frac{V_a}{4} + \frac{V_a}{15} = 0$$

$$6V_a - 60 + 15V_a + 4V_a = 0$$

$$25V_a = 60 \quad , \quad \text{so } V_a = \frac{60}{25} = 2.4$$

Therefore $$i_L(t)\Big|_{t\to\infty} = \frac{2.4}{4} = 0.6\text{ A}$$

Using the assumed solution,

$$i_L(t) = K_1 + K_2 e^{-t/\tau}$$

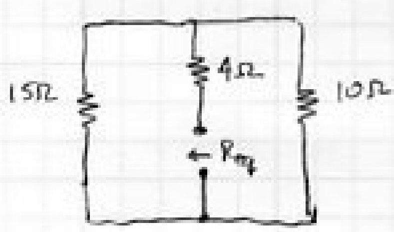
At $t = 0^-$: $\quad i_L(0^-) = 1.25 = K + K_2$

At $t \to \infty$: $\quad i_L(\infty) = 0.6 = K_1$

so $K_1 = 0.6$

$K_2 = 0.65$

And the time constant

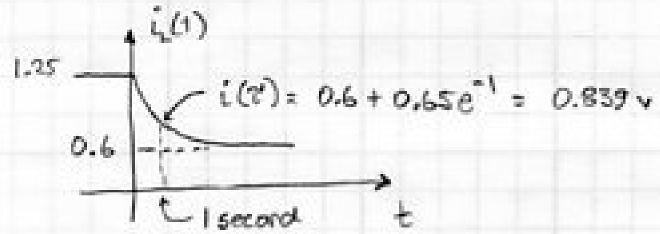


$$R_{eq} = 10 \,/\!/\, 15 + 4$$
$$= 6 + 4$$
$$= 10\,\Omega$$

so $$\tau = \frac{L}{R} = \frac{10\text{ H}}{10\,\Omega} = 1 \text{ second}$$

Finally, $$i_L(t) = 0.6 + 0.65e^{-t}$$

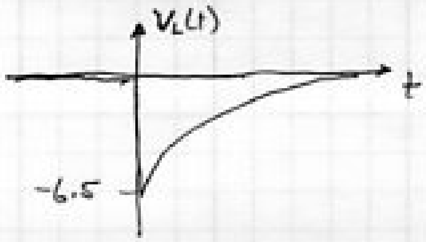
Sketch :



(b) Need $V_L(t)$, so we may use $V_L(t) = L\dfrac{di_L(t)}{dt}$

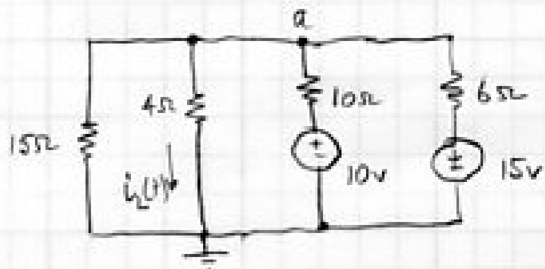
$$\begin{aligned} V_L(t) &= L\frac{d}{dt}\left[0.6 + 0.65e^{-t}\right] \\ &= 10\left[-0.65e^{-t}\right] \\ &= -6.5e^{-t} \end{aligned}$$

Sketch :



(c) At $t = 10s$, $\quad i_L(10) = 0.6 + 0.65e^{-10}$
$\qquad = 0.60003 \approx 0.6$

At $t \to \infty$, circuit becomes



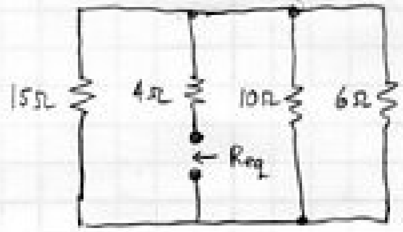
At node a, $\quad \dfrac{V_a}{15} + \dfrac{V_a}{4} + \dfrac{V_a - 10}{10} + \dfrac{V_a - 15}{6} = 0$

$$4V_a + 15V_a + 6V_a - 60 + 10V_a - 150 = 0$$

$$35V_a = 210$$
$$V_a = 6\text{ V}$$

so $i_L(t)\Big|_{t\to\infty} = \dfrac{6}{4} = 1.50$ A

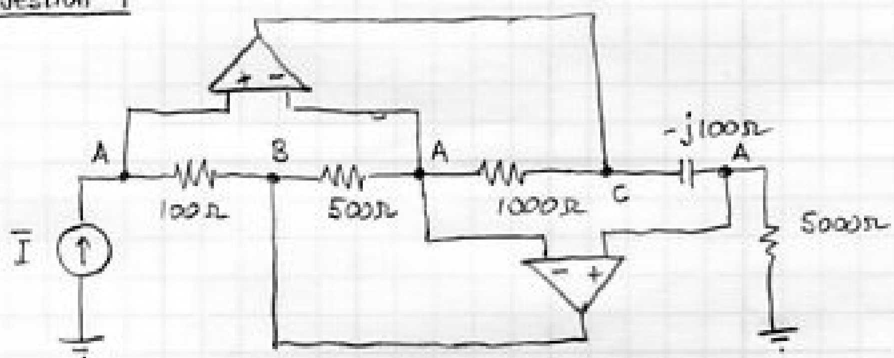
Equivalent resistance when $t > 10s$.



$$R_{eq} = (15 /\!/ 10) /\!/ 6 + 4$$
$$= 6/\!/6 + 4$$
$$= 3 + 4$$
$$= 7\Omega$$

New time constant for $t > 10s$, $\tau = \frac{L}{R} = \frac{10}{7}$ seconds

Question 4



Note that the summing point constraints force three nodes to have the same voltage V_A.

At left-most node A: $$-\bar{I} + i_p + \frac{\bar{V}_A - \bar{V}_B}{100} = 0$$

so $$\bar{V}_A - \bar{V}_B = 100\,\bar{I} \qquad (1)$$

At middle node A: $$\frac{\bar{V}_A - \bar{V}_B}{500} + \frac{\bar{V}_A - \bar{V}_C}{1000} + i_p + i_n = 0$$

$$2\bar{V}_A - 2\bar{V}_B + \bar{V}_A - \bar{V}_C = 0$$
$$3\bar{V}_a - 2\bar{V}_B - \bar{V}_c = 0 \qquad (2)$$

At right-most node A: $$\frac{\bar{V}_A}{5000} + i_p + \frac{\bar{V}_A - \bar{V}_C}{-j100} = 0$$

$$\bar{V}_A + j50\,\bar{V}_A - j50\bar{V}_c = 0$$
$$\bar{V}_A\,(1 + j50) = j50\,\bar{V}_c \qquad (3)$$

From (1), $\bar{V}_B = \bar{V}_A - 100\,\bar{I}$

Substitute into (2),

$$3\bar{V}_a - 2(\bar{V}_a - 100\bar{I}) - \bar{V}_c = 0$$
$$\bar{V}_a + 200\,\bar{I} = \bar{V}_c$$

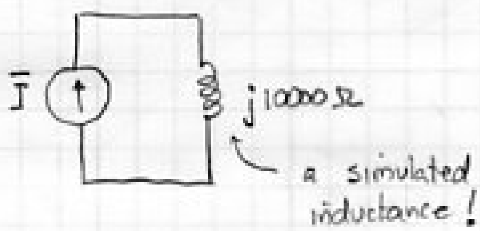
Substitute into (3),

$$\bar{V}_A(1 + j50) = j50(\bar{V}_a + 200\,\bar{I})$$
$$\bar{V}_a + j50\bar{V}_a = j50\bar{V}_a + j10000\,\bar{I}$$
$$\text{so} \quad \bar{V}_a = j10000\,\bar{I}$$

(b) Total impedance $Z_{eq} = \dfrac{\bar{V}_a}{\bar{I}} = \dfrac{j10000\,\bar{I}}{\bar{I}}$

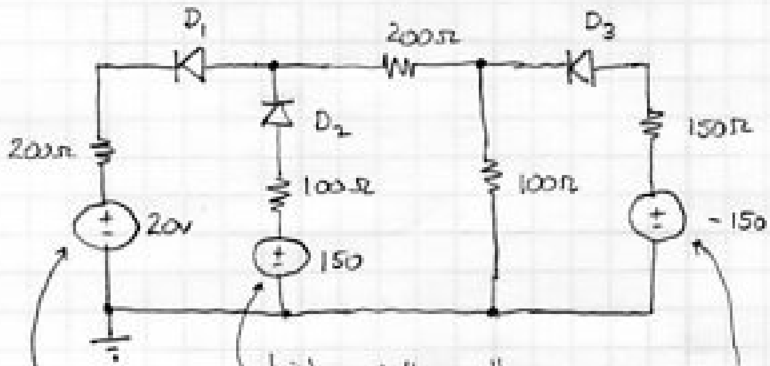
$$= j10000\,\Omega$$

Equivalent circuit



Question 5

(a)



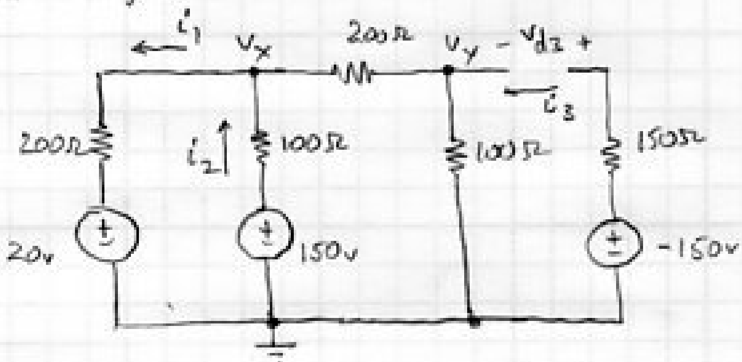
relatively low voltage – looks like D_1 is on. The 150v source looks like it will drive current downward in this branch.

higher voltage than anywhere else in circuit, suggesting that D_2 is on.

large negative voltage, suggesting current <u>downward</u> through the source. Looks like D_3 is off.

D_1 on
D_2 on
D_3 off

Ideal analysis :



At node V_x : $\frac{V_x - 150}{100} + \frac{V_x - 20}{200} + \frac{V_x - V_y}{200} = 0$

$$2V_x - 300 + V_x - 20 + V_x - V_y = 0$$

$$4V_x - V_y = 320 \qquad (1)$$

At node V_y: $\dfrac{V_y - V_x}{200} + \dfrac{V_y}{100} = 0$

$$V_y - V_x + 2V_y = 0$$

$$V_x = 3V_y \qquad (2)$$

Substitute into (1)

$$4(3V_y) - V_y = 320$$
$$12V_y - V_y = 320$$
$$11V_y = 320, \quad \text{so} \quad V_y = \frac{320}{11} = \boxed{29.09\ \text{v}}$$

and $V_x = 3V_y$
$$= \frac{960}{11} = \boxed{87.27\ \text{v}}$$

We have $i_1 = \dfrac{V_x - 20}{200} = 0.336$ A (<u>POSITIVE</u> - good!)

$i_2 = -\dfrac{(V_x - 150)}{100} = 0.626$ A (<u>POSITIVE</u> - good!)

$i_3 = 0$

→ check D_3 voltage $-150 - V_y = -179.09$ v
(<u>NEGATIVE</u> - good!)

(b) Need diode models

Segment A: slope = $10\text{mA}/(2.7 - 0.7) = 0.005$
so $R_A = 1/0.005 = 200\Omega$

intercept precisely at 0.7v, so

$$V = 200i + 0.7$$

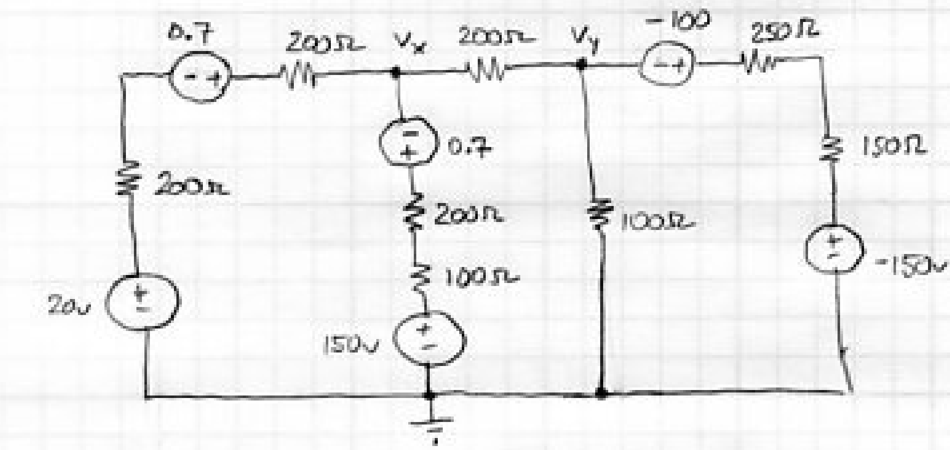
Segment B: $i_d = 0$, so B is an <u>open circuit</u>

Segment C: slope = $20\text{mA}/(-100+105) = 0.004$
so $R_c = 250\,\Omega$

intercept at −100v, so

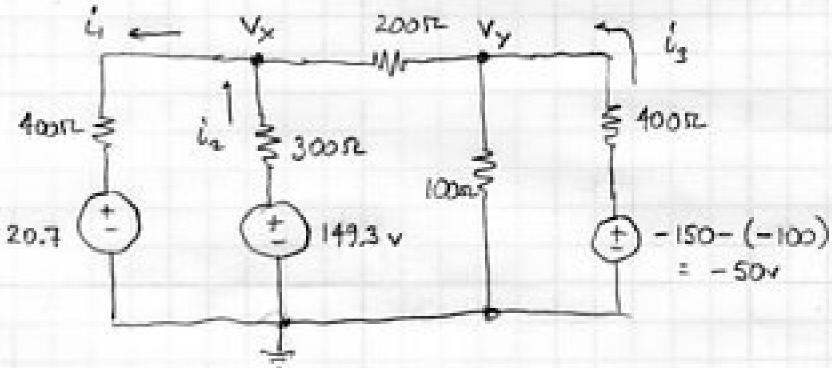
$v = 250i - 100$

Analyze circuit



D_1, D_2 → segment A
D_3 → operating in breakdown, segment C.

Redraw circuit



At V_x: $\frac{V_x - 20.7}{400} + \frac{V_x - 149.3}{300} + \frac{V_x - V_y}{200} = 0$

$$3V_x - 62.1 + 4V_x - 597.2 + 6V_x - 6V_y = 0$$

$$13V_x - 6V_y = 659.3 \qquad (1)$$

At V_y: $\frac{V_y - V_x}{200} + \frac{V_y}{100} + \frac{V_y + 50}{400} = 0$

$$2V_y - 2V_x + 4V_y + V_y + 50 = 0$$

$$-2V_x + 7V_y = -50 \qquad (2)$$

From (2), $-2V_x = -50 - 7V_y$, so $V_x = 25 + \frac{7}{2}V_y$
Substitute into (1)

$$13\left(25 + \frac{7}{2}V_y\right) - 6V_y = 659.3$$

$$325 + \frac{91}{2}V_y = 659.3$$

$$40.5\,V_y = 334.3$$

so $V_y = \boxed{8.254\text{ v}}$

And $V_x = 25 + \frac{7}{2}V_y = \boxed{53.89\text{ v}}$

And $i_1 = \frac{V_x - 20.7}{400} = 0.06812\text{ A}$

$\rightarrow v = (0.06812) \times 200 + 0.7 = 14.32\text{ v}$
[confirmed on segment A]

$i_2 = \frac{-(V_x - 149.3)}{300} = 0.318\text{ A}$

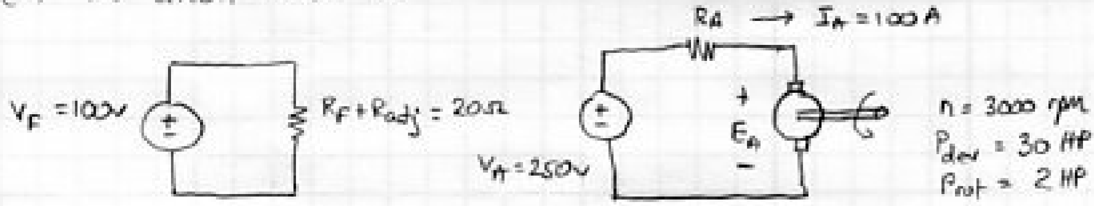
$\rightarrow v = (0.318) \times 200 + 0.7 = 64.307\text{ v}$
[confirmed on segment A]

$i_3 = \frac{-50 - V_y}{400} = -0.1456\text{ A}$

$\rightarrow v = (-0.1456) \times 250 - 100 = -136.41\text{ v}$
[confirmed on segment C]

Question 6

(a) The circuit model ...



- The armature EMF E_A

We have $P_{dev} = I_A E_A$, so $E_A = \dfrac{P_{dev}}{I_A}$

$$E_A = \frac{30 \times 746\ W}{100} = 223.8\ v$$

- Armature resistance R_A

$$V_A = E_A + I_A R_A, \quad \text{so } R_A = \frac{V_A - E_A}{I_A}$$

$$R_A = \frac{250 - 223.8}{100} = 0.262\,\Omega$$

- Machine constant $K\phi$

Can use $E_A = K\phi\,\omega_m$, so $K\phi = \dfrac{E_A}{\omega_m}$

$$K\phi = \frac{223.8}{3000 \times 2\pi/60} = 0.71238$$

- Efficiency η.

Total output power $P_{out} = P_{dev} - P_{rot}$
$= 28$ HP
$= 20888$ W

Total electrical input power $P_{in} = V_A I_A + \dfrac{V_F^2}{(R_F + R_{adj})}$

$$P_{in} = 250 \times 100 + \frac{100^2}{20}$$
$$= 25500\ W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{20888}{25500} \times 100\%$$
$$= 81.91\ \%$$

(b) Motor is run with 0 developed power. This means that developed torque $T_{dev} = 0$, and

$$T_{dev} = k\phi I_A , \quad \text{so} \quad I_A = 0.$$

Therefore, $E_A = V_A = 250\text{ V}$.

And $E_A = k\phi\omega_m$, so $\omega_m = \frac{E_A}{k\phi}$

$$\omega_m = \frac{250}{0.71238} = 350.936 \text{ r/s}$$

so $n = 350.936 \times 60 / 2\pi$

$n = 3351.19$ rpm (speeds up!)

Wow! 15 pages.