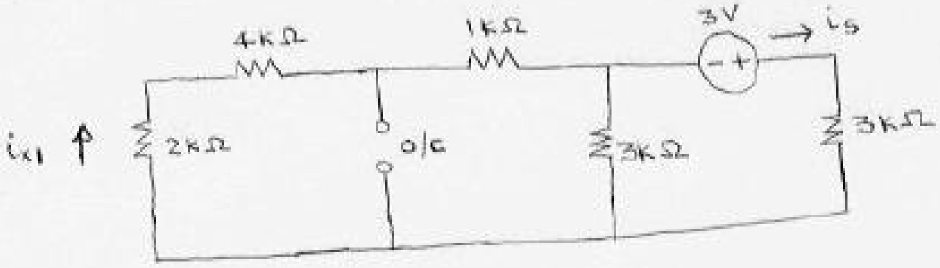
Mid-Term Solutions

Q1. Step #1: consider 3V source only.



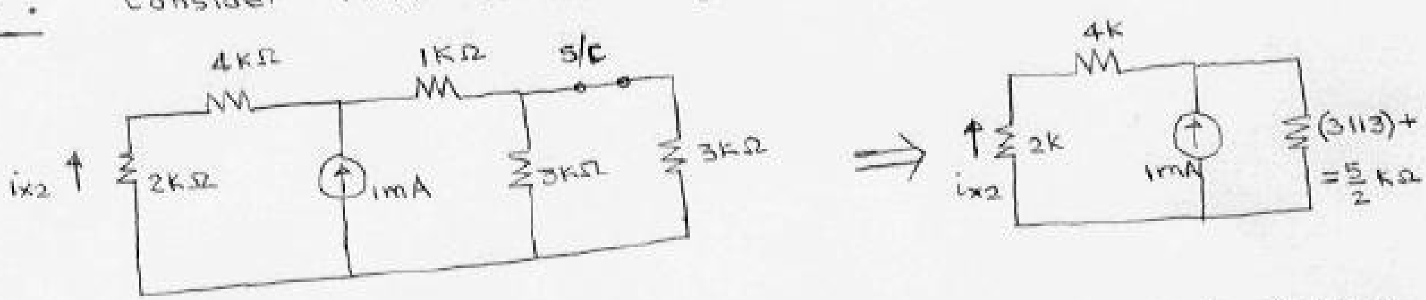
R_{eq} seen by 3V source $= (7K \parallel 3K) + 3K = \frac{51}{10}\ k\Omega$

Total current supplied by the source $i_s = \frac{3V}{R_{eq}} = \frac{3\ V}{\frac{51}{10}\ k\Omega}$

$= \frac{30}{51}\ mA$

Current Divider gives $i_{x1} = i_s * \frac{3\ K}{3K + 7K} = \frac{30}{51} \times \frac{3}{10} = \frac{3}{17}\ mA$

Step #2: Consider 1mA source only.



$i_{x2} = -1\ mA * \frac{5/2\ k\Omega}{5/2\ k\Omega + 6\ k\Omega} = \frac{-5}{17}\ mA$ using current division.

Step #3: Apply principle of superposition.

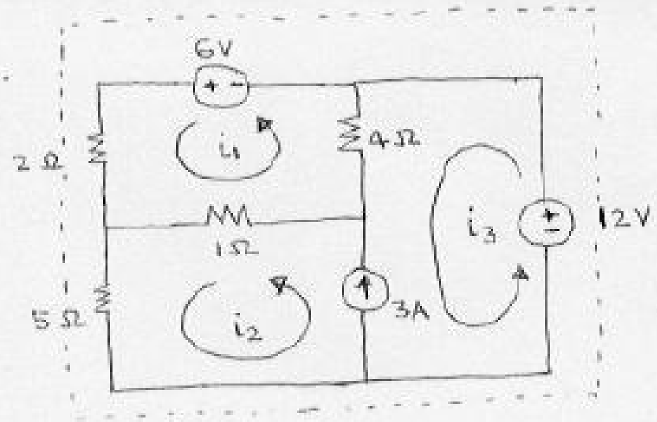
$$i_x = i_{x1} + i_{x2}$$

$$= \frac{3}{17}\ mA + \left(-\frac{5}{17}\ mA\right)$$

$$= \frac{-2}{17}\ mA$$

In decimals, $i_x = -0.1176\ mA$

Q2.



KVL in loop 1 :

$$-6 + 2i_1 + 1(i_1 - i_2) + 4(i_1 - i_3) = 0$$

i.e., $7i_1 - i_2 - 4i_3 = 6 \rightarrow ①$

observation :

$i_2 - i_3 = 3 \rightarrow ②$

KVL for outer loop shown by dotted line :

$$-12 - 6 + 2i_1 + 5i_2 = 0$$

i.e., $2i_1 + 5i_2 = 18 \rightarrow ③$

solve ①, ② and ③ to find i_3 supplied by 12-V source.

$7i_1 - (i_3 + 3) - 4i_3 = 6$

$7i_1 - 5i_3 = 9 \rightarrow ④$

$2i_1 + 5(i_3 + 3) = 18$

$2i_1 + 5i_3 = 3 \rightarrow ⑤$

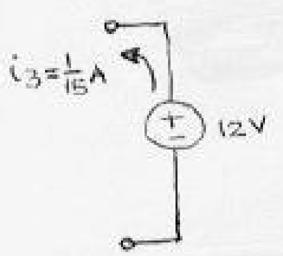
④ × 2 $\quad 14i_1 - 10i_3 = 18$

⑤ × 7 $\quad 14i_1 + 35i_3 = 21$

subtract $\quad 0 - 45i_3 = -3$

$$i_3 = \frac{1}{15}\text{ A}$$

Power absorbed or delivered = ? Consider 12-V source.



∴ By convention, power is delivered.

$$P_{12V} = 12 * \frac{1}{15} = \frac{4}{5}\text{ W}$$

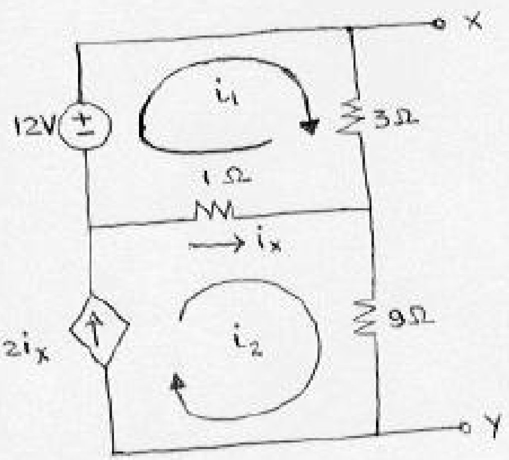
In decimals, $P_{12V} = 0.8$ W

Q3.

Step #1 : Find open-circuit voltage V_{xy}.

Define node Y as reference node.

$\therefore V_{xy} = V_x - V_y = V_x - 0 = V_x$. V_x needs to be calculated.



observation : $i_2 = 2i_x \rightarrow (1)$

Observation : $i_2 - i_1 = i_x \rightarrow (2)$

Substituting (1) in (2) gives

$$i_2 - i_1 = \frac{i_2}{2}$$

$$-2i_1 + i_2 = 0 \rightarrow (3)$$

KVL in loop 1 gives :

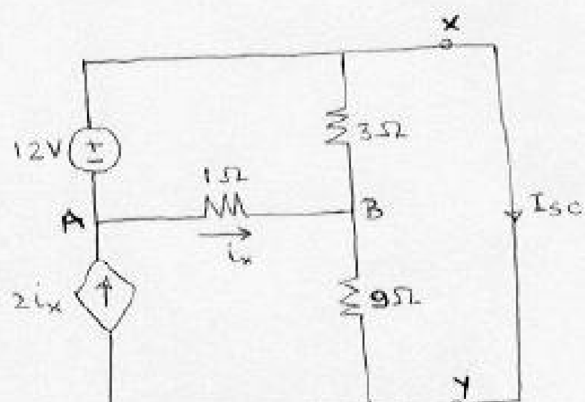
$$-12 + 3i_1 + 1(i_1 - i_2) = 0$$

i.e., $4i_1 - i_2 = 12 \rightarrow (4)$

Solving (3) and (4) gives $i_1 = 6A$ and $i_2 = 12A$.

$$\therefore V_{xy} = V_x = 3i_1 + 9i_2 = 3 * 6 + 9 * 12 = 126 V$$

Step #2 : Find short-circuit current I_{sc}. Consider node Y as reference node i.e. $V_y = 0V$. Also, $V_x = 0V$.



From observation,

$$V_x - V_A = 12 V$$

$$\therefore 0 - V_A = 12$$

$$V_A = -12 V$$

V_B needs to be calculated.

KCL at node B gives :

$$\frac{V_B - (-12)}{1} + \frac{V_B - 0}{3} + \frac{V_B - 0}{9} = 0$$

$$V_B = -\frac{108}{13} \text{ V}$$

$$\therefore i_x = \frac{V_A - V_B}{1} = \frac{-12 - \left(\frac{-108}{13}\right)}{1} = -\frac{48}{13} \text{ A}$$

KCL at node Y gives :

$$I_{sc} = 2 i_x + \frac{0 - V_B}{9} = -\frac{96}{13} + \frac{12}{13} = \frac{-84}{13} \text{ A}$$

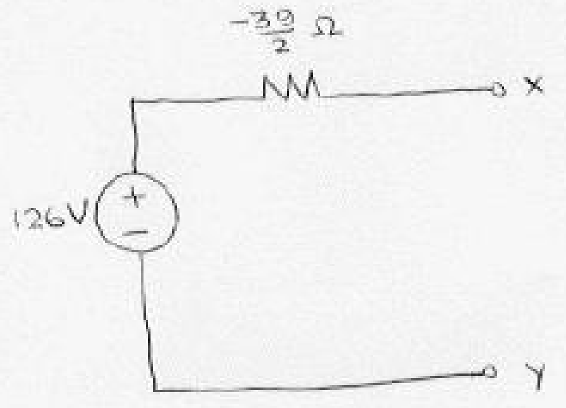
Step #3 :

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{V_{XY}}{I_{sc}} = \frac{126}{-84/13} = -\frac{39}{2} \ \Omega$$

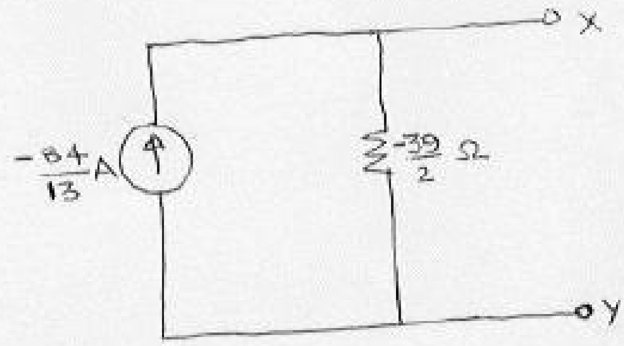
Too bad R_{TH} works out negative.

Step #4 :

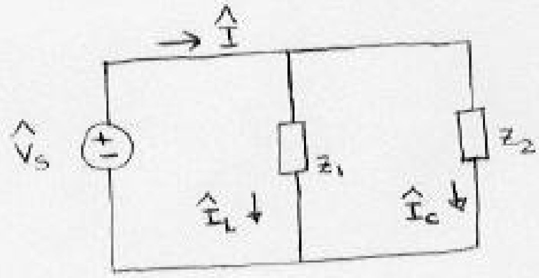
Thevenin Equivalent Circuit



Norton Equivalent Circuit



Q4.



$$Z_1 = Z_L = j\omega L = j * 377 * 10 \times 10^{-3} = j\,3.77\ \Omega = 3.77\angle 90^\circ\ \Omega$$

$$Z_2 = Z_R + Z_C = R - \frac{j}{\omega C} = 30 - j * \frac{1}{(377 \times 30 * 10^{-6})} = 30 - j\,88.417 = 93.368\angle -71.258^\circ$$

$$\hat{I}_L = \frac{\hat{V}_s}{Z_1} = \frac{120\angle -45^\circ}{3.77\angle 90^\circ} = 31.83\angle -135^\circ\ \text{A}$$

$$= -22.507 - j\,22.507\ \text{A}$$

$$\hat{I}_c = \frac{\hat{V}_s}{Z_2} = \frac{120\angle -45^\circ}{93.368\angle -71.258^\circ} = 1.285\angle 26.258^\circ\ \text{A}$$

$$= 1.1524 + j\,0.5685\ \text{A}$$

$\therefore$ Applying KCL $\hat{I} = \hat{I}_L + \hat{I}_c$

$$= -22.507 - j\,22.507 + 1.1524 + j\,0.5685$$

$$= -21.3546 - j\,21.9385\ \text{A}$$

$$= 30.6156\angle -134.2273^\circ\ \text{A}$$

$$\simeq 30.62\angle -134.23^\circ\ \text{A}$$

$$i(t) = 30.62\cos(377t - 134.23^\circ)\ \text{A}$$

Note : Either 1- or 2-digit precision is good in phasor analysis questions.