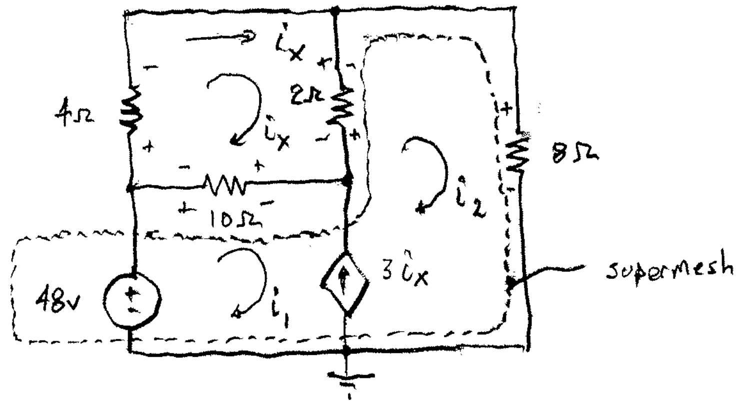
Question 1

(a)



Mesh i_x:
$$4i_x + 2(i_x - i_2) + 10(i_x - i_1) = 0$$
$$4i_x + 2i_x - 2i_2 + 10i_x - 10i_1 = 0$$

$$16i_x - 10i_1 - 2i_2 = 0 \qquad (1)$$

Supermesh:
$$-48 + 10(i_1 - i_x) + 2(i_2 - i_x) + 8i_2 = 0$$
$$-48 + 10i_1 - 10i_x + 2i_2 - 2i_x + 8i_2 = 0$$

$$10i_1 + 10i_2 - 12i_x = 48 \qquad (2)$$

And inside the supermesh, $i_2 - i_1 = 3i_x$

so $i_2 = i_1 + 3i_x \qquad (3)$

Substitute (3) into (1) and (2)

(1)
$$16i_x - 10i_1 - 2(i_1 + 3i_x) = 0$$
$$10i_x - 12i_1 = 0$$

so $i_x = \frac{6}{5} i_1 \qquad (4)$

(2)
$$10i_1 + 10(i_1 + 3i_x) - 12i_x = 48$$
$$20i_1 + 18i_x = 48 \qquad (5)$$

Substitute (4) into (5)

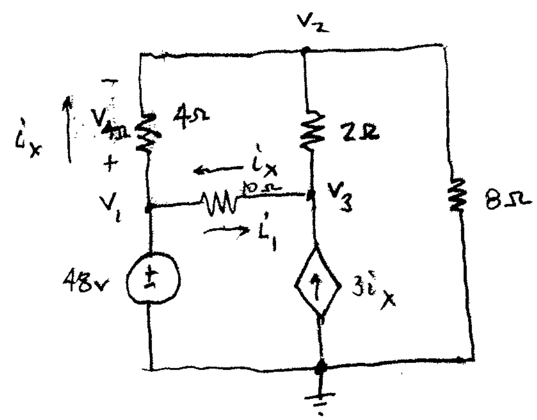
$$20i_1 + \frac{108}{5} i_1 = 48$$

$$\frac{208}{5} i_1 = 48 \qquad \therefore \boxed{i_1 = 1.1538 \text{ A}}$$

And $\boxed{i_x = \frac{6}{5} i_1 = 1.3846}$

And, from (3), $\boxed{i_2 = i_1 + 3 i_x = 5.3077 \text{ A}}$

(b)



V_1 already known $\boxed{V_1 = 48 \text{ v}}$

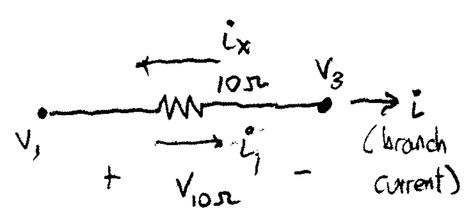
We have $V_{4\Omega} = 4 i_x = 5.5385$ v. By inspection, we may write

$$V_1 - V_2 = 5.5385 \text{v},$$

so $V_2 = V_1 - 5.5385$
$= 48 - 5.5385$

$$\boxed{V_2 = 42.4615 \text{ v}}$$

Between V_1 and V_3, we have



we have $i = i_1 - i_x$
$= -0.23077$ A

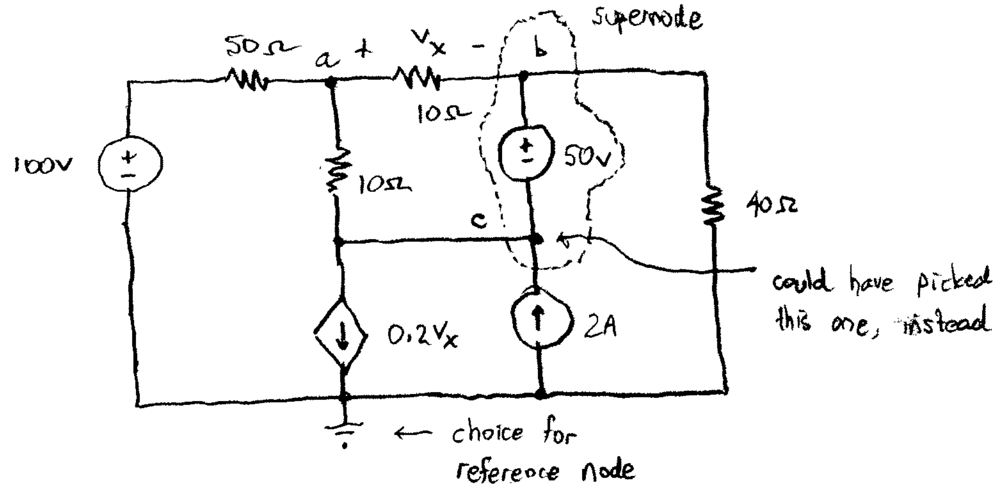
so $V_{10\Omega} = 10 i = -2.3077$ V

And, by inspection,

$V_1 - V_3 = V_{10\Omega}$ so $V_3 = V_1 - V_{10}$
$= 48 + 2.3077$

$$\boxed{V_3 = 50.3077}$$

Question 2



(a) Node voltages:

Node a:
$$\frac{V_a - 100}{50} + \frac{V_a - V_c}{10} + \frac{V_a - V_b}{10} = 0$$

$$V_a - 100 + 5V_a - 5V_c + 5V_a - 5V_b = 0$$

$$11V_a - 5V_b - 5V_c = 100 \qquad (1)$$

Supernode:
$$\frac{V_b - V_a}{10} + \frac{V_b}{40} + \frac{V_c - V_a}{10} - 2 + 0.2V_x = 0$$

$$4V_b - 4V_a + V_b + 4V_c - 4V_a - 80 + 8V_x = 0$$

$$5V_b - 8V_a + 4V_c + 8V_x = 80 \qquad (2)$$

We have the dependency $V_x = V_a - V_b$, so equation (2) becomes

$$5V_b - 8V_a + 4V_c + 8(V_a - V_b) = 80$$

$$5V_b - 8V_a + 4V_c + 8V_a - 8V_b = 80$$

$$-3V_b + 4V_c = 80 \qquad (3)$$

Within the supernode, $V_b - V_c = 50\,V$

$$\text{so } V_b = V_c + 50 \qquad (4)$$

Substitute (4) into (3)

$$-3(V_c + 50) + 4V_c = 80$$
$$-3V_c - 150 + 4V_c = 80$$

$$\therefore \boxed{V_c = 80 + 150 = 230\,\text{v}}$$

And, from (4), $\boxed{V_b = V_c + 50 = 280\,\text{v}}$

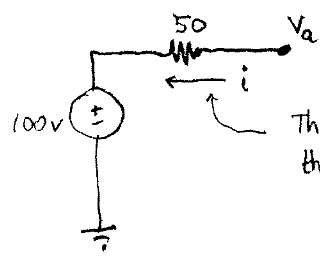
Finally, from (1),

$$11V_a - 5(280) - 5(230) = 100$$

$$11V_a = 2650$$

$$\therefore \boxed{V_a = 240.91\,\text{v}}$$

(b) Consider node a; we need the current in the 100 v source.



This is the first term in the node equation (1) for node a.

$$i = \frac{V_a - 100}{50} = 2.818\ \text{A}$$

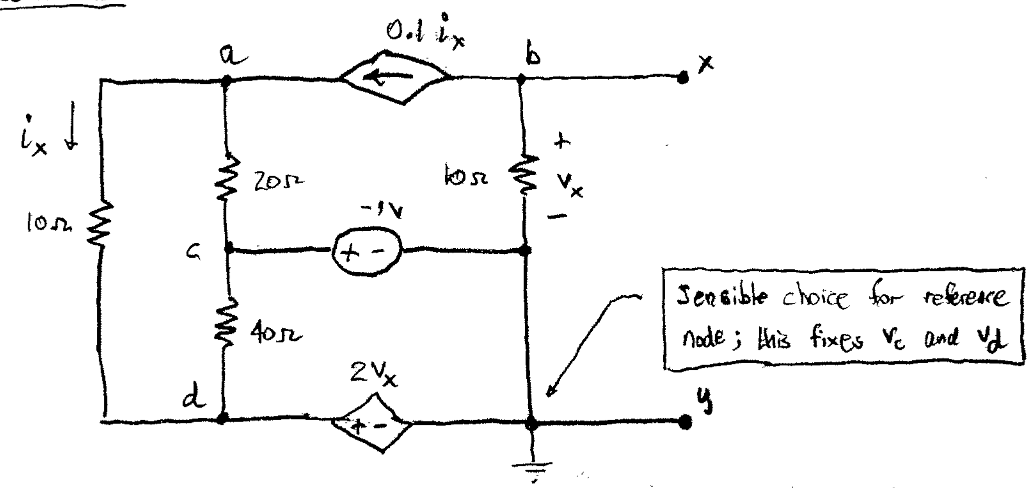
Observing the passive reference convention,

$$P_{100} = +Vi = 100 \times 2.818$$

so $\boxed{P_{100} = 281.8\ \omega}$ POSITIVE IN VALUE, SO POWER IS <u>ABSORBED</u>.

Question 3

(a)



Set up node voltage method to find Thevenin voltage V_t.

By inspection, $V_c = -1V$
$V_d = 2V_x$

and we note that $V_x = 2V_b$.

Node a: $\quad -0.1\,i_x + \dfrac{V_a - V_c}{20} + \dfrac{V_a - V_d}{10} = 0$

We observe that $i_x = \dfrac{V_a - V_d}{10}$, so

$$-0.1\left(\frac{V_a - V_d}{10}\right) + \frac{V_a - V_c}{20} + \frac{V_a - V_d}{10} = 0$$

$(\times 100)$ $\quad V_d - V_a + 5V_a - 5V_c + 10V_a - 10V_d = 0$

$$14V_a - 5V_c - 9V_d = 0$$

or $\quad 14V_a - 5(-1) - 9(2V_b) = 0$

$$14V_a - 18V_b = -5 \qquad (1)$$

Node b: $\quad 0.1\,i_x + \dfrac{V_b}{10} = 0$

so $\quad 0.1\left(\dfrac{V_a - V_d}{10}\right) + \dfrac{V_b}{10} = 0$

or $\quad \dfrac{V_a - 2V_b}{100} + \dfrac{V_b}{10} = 0$

$$V_a - 2V_b + 10V_b = 0$$

$$\text{so} \quad V_a = -8V_b \qquad (2)$$

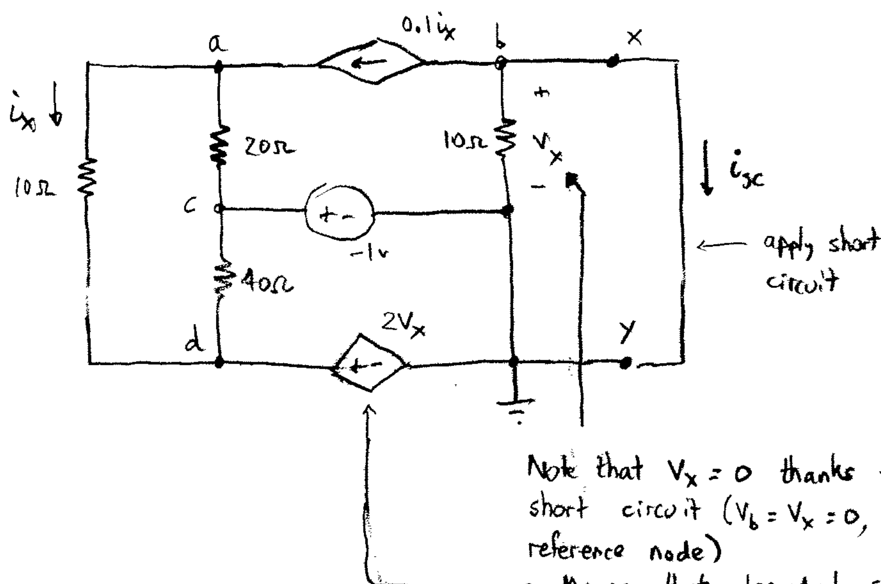
We need V_b, which is V_t. Substitute (2) into (1)

$$14(-8V_b) - 18V_b = -5$$

$$-130V_b = -5$$

$$\therefore \boxed{V_t = V_b = \frac{5}{130} = \frac{1}{26}\text{ V} = 38.462\text{ mV}}$$

Now, the Thevenin resistance R_t. Unfortunately, we can't take any short-cuts, and must determine short-circuit current i_{sc}.



Note that $V_x = 0$ thanks to short circuit ($V_b = V_x = 0$, same as reference node)
- Means that dependent source is a short circuit!

Node a: $$\frac{V_a}{10} + \frac{V_a + 1}{20} - 0.1\, i_x = 0$$

where $i_x = \frac{V_a}{10}$, so $$\frac{V_a}{10} + \frac{V_a+1}{20} - \frac{V_a}{100} = 0$$

$$10V_a + 5(V_a + 1) - V_a = 0$$

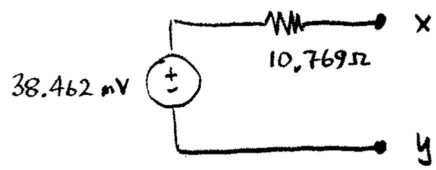
$$14V_a = -5 \qquad \therefore V_a = \frac{-5}{14}$$

Node b: $\quad \cancelto{0}{\frac{V_b}{10}} + 0.1\, i_x + i_{sc} = 0$

$$0.1\left(\frac{V_a}{10}\right) + i_{sc} = 0$$

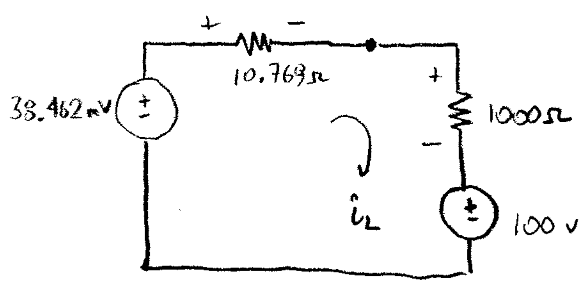
so $\quad i_{sc} = -\frac{V_a}{100} = \frac{5}{1400}\text{ A} = 3.5714 \text{ mA}$

Finally, $\boxed{R_t = \frac{V_t}{i_{sc}} = \frac{1/26}{5/1400} = \frac{280}{26}\,\Omega = 10.769\,\Omega}$



THEVENIN EQUIVALENT CIRCUIT

(b) Use the above Thevenin equivalent!

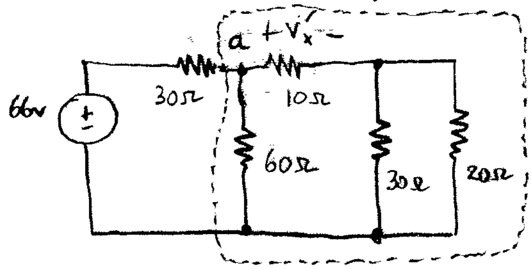


By KVL, $\quad -0.038462 + (10.769 + 1000)\, i_L + 100 = 0$

$$\therefore \quad i_L = -98.897 \text{ mA}$$

Question 4

Consider the 66 V source alone ; zero the others. Find V_x'.



$$R_{eq} = [(20 // 30) + 10] // 60$$
$$= (12 + 10) // 60$$
$$= \frac{22 \times 60}{22 + 60} = 16.0976\ \Omega$$

Based on the above grouping of series-parallel resistors, we can ~~treat~~ it as a simple voltage divider to find V_a

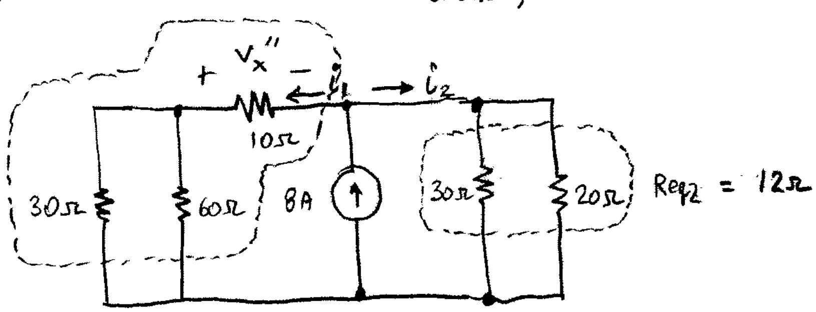
$$V_a = \frac{16.0976}{16.0976 + 30} \times 66 = 23.0476\ v$$

Voltage divider again to find V_x'

$$V_x' = \frac{10}{10 + (30 // 20)} V_a = \frac{10}{22} \times 23.0476$$

so $\boxed{V_x' = 10.476\,v}$

Next, consider the 8 A source alone ; zero all others. Find V_x''



$R_{eq1} = 30 // 60 + 10$
$= 30\ \Omega$

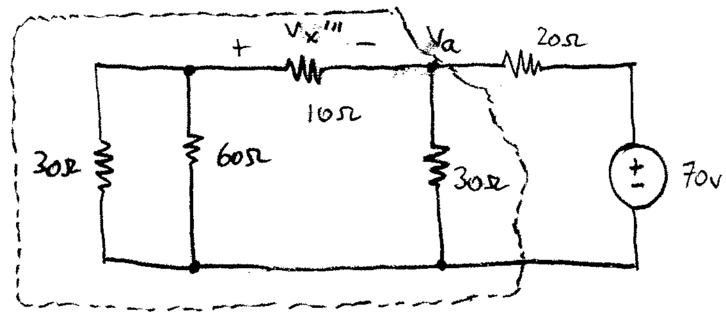
We have a current divider with R_{eq1} and R_{eq2} in parallel. Need i_1

$$i_1 = \frac{R_{eq2}}{R_{eq1} + R_{eq2}} \times 8A$$

$$= \frac{12}{12 + 30} \times 8 = 2.2851\ A$$

Giving $\boxed{V_x'' = -10 i_1 = -22.851 \text{ V}}$

Finally, the 70 V source alone. Find V_x'''



$$Req = [(30 /\!/ 60) + 10] /\!/ 30$$
$$= 15\Omega$$

Following the same analysis as for the 66v source,

$$V_a = \frac{15}{15 + 20} \times 70\text{v} = 30\text{ V}$$

Voltage divider again to find V_x''' (note polarity).

$$V_x''' = \frac{-10}{(60 /\!/ 30) + 10} \times V_a = \frac{10}{30} \times 30$$

$$\boxed{V_x''' = -10\text{v}}$$

Finally, by superposition

$$V_x = V_x' + V_x'' + V_x'''$$
$$= 10.476 - 22.851 - 10$$

$$\boxed{V_x = -22.375 \text{ v}}$$