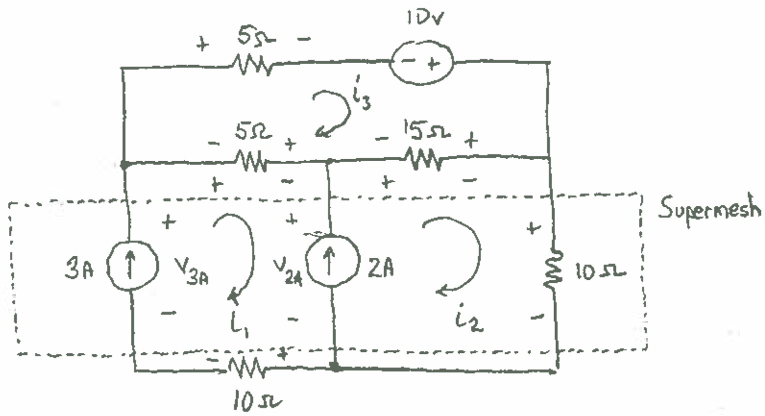
Question 1



(a) We have a supermesh in which i_1 is already known. $i_1 = 3A$.
From the supermesh dependence equation

$$i_2 - i_1 = 2A, \quad \text{so} \quad i_2 = i_1 + 2A = 5A$$

Mesh 3:

$$5i_3 - 10 + 15(i_3 - i_2) + 5(i_3 - i_1) = 0$$

$$5i_3 + 15i_3 - 75 + 5i_3 - 15 = 10$$
$$25i_3 = 100$$

so $i_3 = 4A$

$$\boxed{\begin{aligned} i_1 &= 3A \\ i_2 &= 5A \\ i_3 &= 4A \end{aligned}}$$

(b) KVL in mesh 2 gives: $-V_{2A} + 15(i_2 - i_3) + 10i_2 = 0$

$$-V_{2A} + 15 \times 1 + 10 \times 5 = 0$$
$$V_{2A} = 65\text{ V}$$

Power in the 2-amp source: $P_{2A} = -V_{2A}(i_2 - i_1)$
$= -65 \times 2$
$= -130\text{ W}$ (130 W supplied)

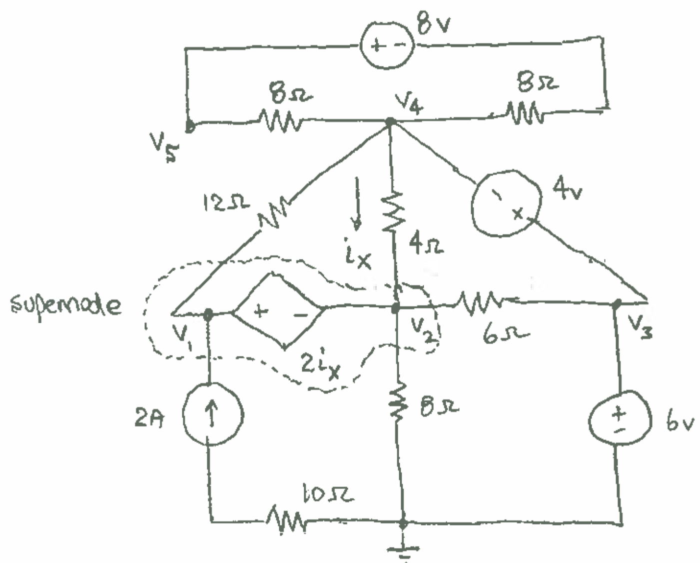
KVL in mesh 1 gives: $-V_{3A} + 5(i_1 - i_3) + V_{2A} + 10i_1 = 0$

$$-V_{3A} + 5 \times -1 + 65 + 10 \times 3 = 0$$
$$V_{3A} = 90\text{ V}$$

Power in the 3-amp source $P_{3A} = -V_{3A}\, i_1$
$= -90 \times 3$
$= -270\text{ W}$ (270 W supplied)

Power in the 10-volt source: $P_{10v} = -10\, i_3$
$= -10 \times 4$
$= -40$ W (40 W supplied)

Question 2



(a) By inspection, $V_3 = 6$ and $V_4 = 6 - 4 = 2$ V.

Supernode equation: $-2 + \dfrac{V_1 - V_4}{12} + \dfrac{V_2}{8} + \dfrac{V_2 - V_4}{4} + \dfrac{V_2 - V_3}{6} = 0$

(x 24) $-48 + 2(V_1 - 2) + 3V_2 + 6(V_2 - 2) + 4(V_2 - 6) = 0$
$-48 + 2V_1 - 4 + 3V_2 + 6V_2 - 12 + 4V_2 - 24 = 0$

$2V_1 + 13V_2 = 88$ (1)

Supernode dependence: $V_1 - V_2 = 2 i_x$ (2)

Dependent source: $i_x = \dfrac{V_4 - V_2}{4} = \dfrac{2 - V_2}{4}$

Combining this with equation (2), $V_1 - V_2 = \dfrac{2 - V_2}{2}$
$2V_1 - 2V_2 = 2 - V_2$
$2V_1 - V_2 = 2$ (3)

Subtract equation (3) from (1):
$2V_1 + 13V_2 = 88$
$-(2V_1 - V_2 = 2)$

$14V_2 = 86$

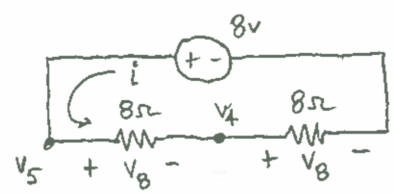
$$\therefore \quad V_2 = \frac{86}{14} = \frac{43}{7} = 6.143 \text{ v.}$$

From equation (3), $\quad 2V_1 = 2 + V_2$

$$2V_1 = 2 + \frac{43}{7} = \frac{57}{7}$$

$$\therefore \quad V_1 = \frac{57}{14} = 4.071 \text{ v}$$

(b) In the top part of the circuit



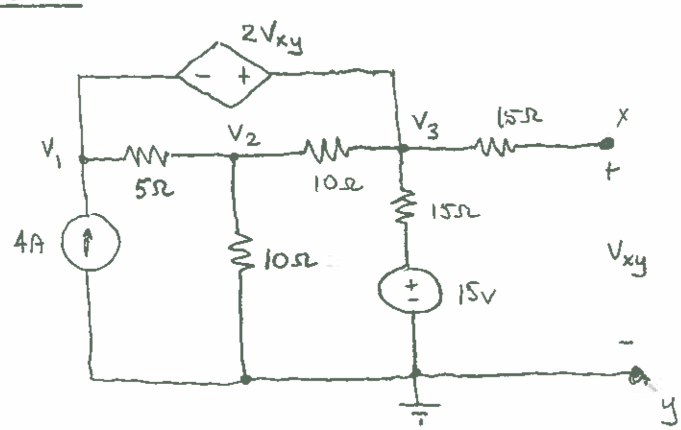
This is a simple voltage divider with $V_8 = \frac{1}{2} \times 8$ v.

$$V_8 = 4\text{V}.$$

We also have $\quad V_5 - V_4 = V_8 = 4$v.

From part (a), $\quad V_4 = 2$v, $\quad$ so $\quad V_5 = 2 + 4$
$= 6$ v

Question 3



(a) Thevenin voltage.

Supernode equation: $\quad -4 + \frac{V_1 - V_2}{5} + \frac{V_3 - V_2}{10} + \frac{V_3 - 15}{15} = 0$

$$(\times 30) \qquad -120 + 6(V_1 - V_2) + 3(V_3 - V_2) + 2(V_3 - 15) = 0$$

$$6V_1 - 6V_2 + 3V_3 - 3V_2 + 2V_3 - 30 = 120$$

$$6V_1 - 9V_2 + 5V_3 = 150 \qquad (1)$$

Supernode dependence: $V_3 - V_1 = 2V_{xy}$

With terminals x and y open-circuited, $V_{xy} = V_3$, so

$$V_3 - V_1 = 2V_3$$

$$V_1 = -V_3 \qquad (2)$$

Node V_2: $\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - V_3}{10} = 0$

(× 10) $2(V_2 - V_1) + V_2 + V_2 - V_3 = 0$

$2V_2 - 2V_1 + V_2 + V_2 - V_3 = 0$

$$-2V_1 + 4V_2 - V_3 = 0 \qquad (3)$$

Substitute equation (2) into (1) and (3)

$-6V_3 - 9V_2 + 5V_3 = 150$

$$-9V_2 - V_3 = 150 \qquad (4)$$

$2V_3 + 4V_2 - V_3 = 0$

$$4V_2 + V_3 = 0 \qquad (5)$$

Add (4) and (5): $4V_2 + V_3 = 0$

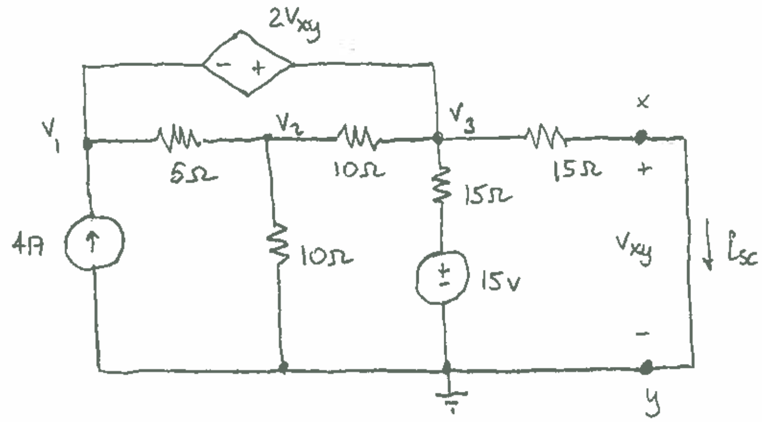
$-9V_2 - V_3 = 150$

$-5V_2 = 150$, so $V_2 = -30$ v.

And, from (3), $V_3 = -4V_2 = 120$ v

Therefore, $\boxed{V_t = V_{oc} = V_3 = 120\text{v}}$

Now Thevenin resistance. Unfortunately, there is a dependent source, so we must determine the short-circuit current i_s.



With the short circuit applied, $V_{xy} = 0$, so the dependent voltage source becomes a short circuit, so $V_3 = V_1$.

Supernode equation: $-4 + \frac{V_1 - V_2}{5} + \frac{V_3 - V_2}{10} + \frac{V_3 - 15}{15} + \frac{V_3}{15} = 0$ This is i_{sc}

(x30) $-120 + 6V_1 - 6V_2 + 3V_3 - 3V_2 + 2V_3 - 30 + 2V_3 = 0$

$$6V_1 - 9V_2 + 7V_3 = 150$$

With $V_1 = V_3$, $6V_3 - 9V_2 + 7V_3 = 150$

$$-9V_2 + 13V_3 = 150 \qquad (1)$$

Node V_2 (equation unchanged): $-2V_1 + 4V_2 - V_3 = 0$

so $-2V_3 + 4V_2 - V_3 = 0$

$$4V_2 - 3V_3 = 0 \qquad (2)$$

From (2), $V_2 = \frac{3}{4}V_3$, so (1) becomes

$$-9\left(\tfrac{3}{4}V_3\right) + 13V_3 = 150$$

(x4) $-27V_3 + 52V_3 = 600$

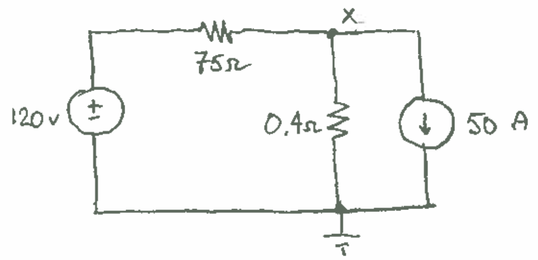
$$25V_3 = 600$$

so $V_3 = 24\text{V}$

Therefore $\boxed{i_{sc} = \frac{V_3}{15} = \frac{24}{15} = \frac{8}{5} = 1.6 \text{ A}}$

Finally, $\boxed{R_t = \frac{V_t}{i_{sc}} = \frac{120}{8/5} = 75\Omega}$

(b) Using the Thevenin equivalent,



At node x : $\dfrac{V_x - 120}{75} + 50 + \dfrac{V_x}{0.4} = 0$

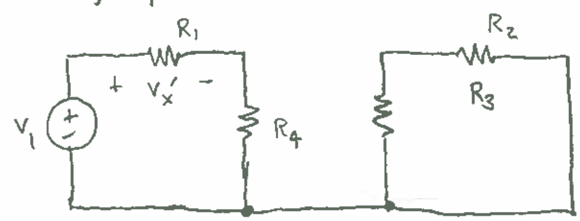
$(\times 75) \quad V_x - 120 + 3750 + 187.5 V_x = 0$

$188.5 V_x = 3630$

$V_x = 19.26 \text{ v.}$

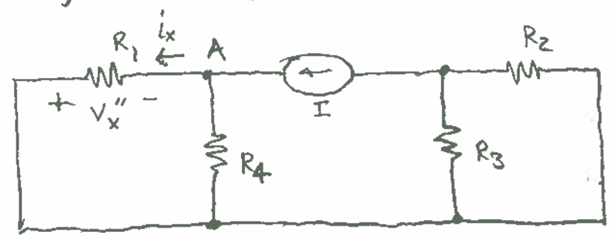
Question 4

(a) With only V_1 active :



This is a simple voltage divider : $V_x' = \dfrac{R_1}{R_1 + R_4} \cdot V_1$

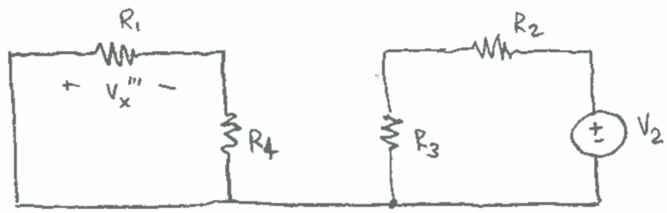
(b) With only I active,



At node A, we have current division : $i_x = \dfrac{R_4}{R_1 + R_4} \cdot I$

and $V_x'' = -i_x R_1 = \dfrac{-R_1 R_4}{R_1 + R_4} \cdot I$

(c) With only V_2 active



No current flow is possible in R_1, so

$$V_x''' = 0$$

Therefore, by superposition:

$$V_x = V_x' + V_x'' + V_x'''$$

$$= V_1\left(\frac{R_1}{R_1+R_4}\right) - I\left(\frac{R_1R_4}{R_1+R_4}\right)$$

$$= \frac{R_1}{R_1+R_4}\left(V - IR_4\right)$$