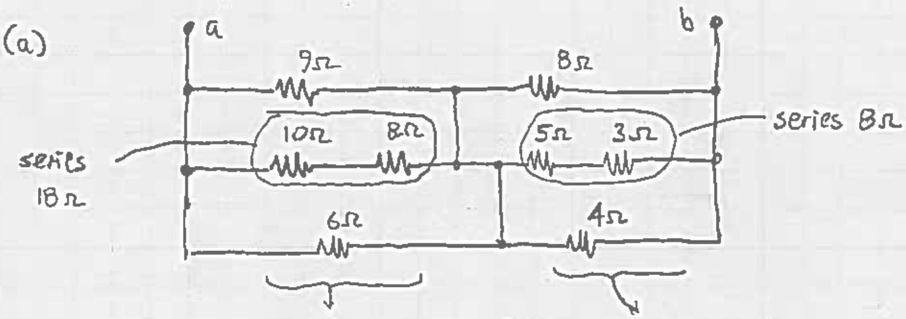
Question 1

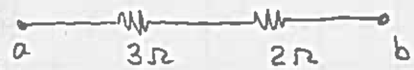
(a)



parallel $\left[\frac{1}{9} + \frac{1}{18} + \frac{1}{6}\right]^{-1} = 3\Omega$

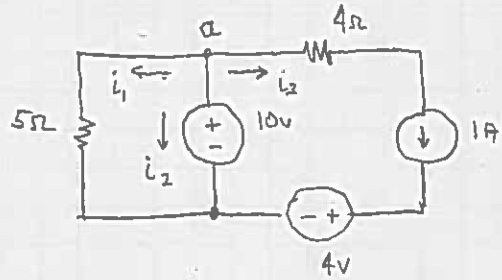
parallel $\left[\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right]^{-1} = 2\Omega$

Simplifying to



$$\boxed{R_{eq} = 5\Omega}$$

(b)



We have $i_3 = 1A$.

Also, $i_1 = \frac{10v}{5\Omega} = 2A$

Then, KCL at node a gives $-i_1 - i_2 - i_3 = 0$

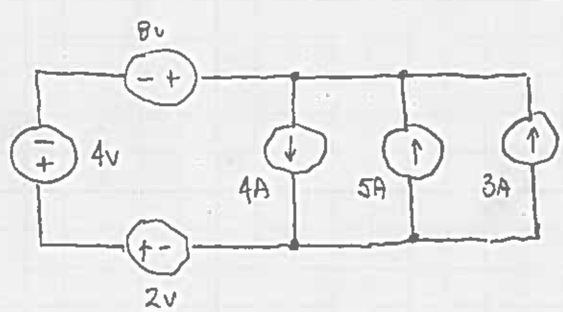
so $i_2 = -i_1 - i_3 = -2 - 1 = -3A$

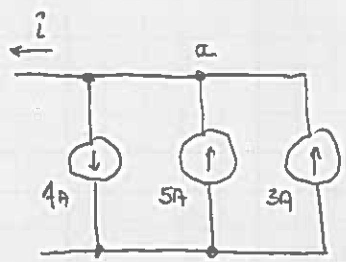
Power in the 10-volt source, using the passive reference convention

$$P_{10v} = vi = 10 \times -3 = -30\ W$$

$$\boxed{P_{10v} = -30\ W}$$

(c)



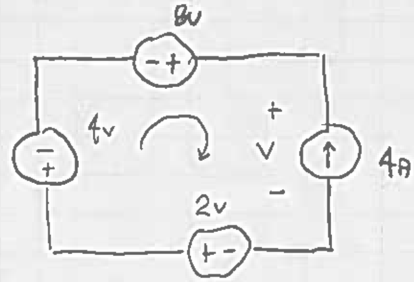


By KCL at node

$-i - 4 + 5 + 3 = 0$

so $i = 4$ A

Simplifies to

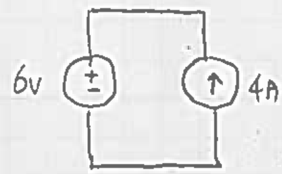


KVL gives

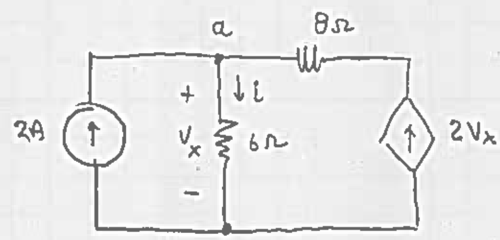
$-2 + 4 - 8 + V = 0$

so $V = 6$v

Equivalent circuit:



(d)



KCL at node a

$2 + 2V_x - i = 0$

$i = 2 + 2V_x \quad (1)$

and $V_x = 6i \quad (2)$

Substituting (2) into (1) gives $i = 2 + 2(6i)$

$= 2 + 12i$

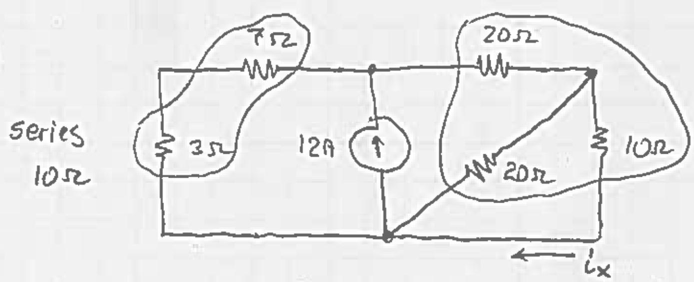
so $-11i = 2$, and $i = -\frac{2}{11}$ A

$= -0.18182$ A

Finally, $V_x = 6i$

$\boxed{V_x = -\frac{12}{11}\text{ V} = -1.09091\text{ v}}$

(e)

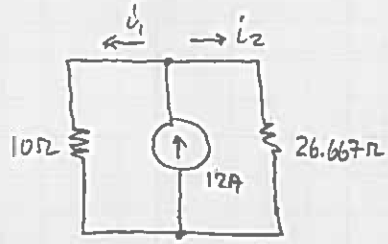


equivalent resistance

$(10 /\!/ 20) + 20$

$= 26.667\Omega$

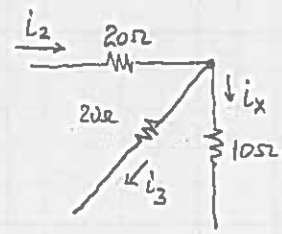
This simplifies to:



This is a current divider, and we need i_2

$$i_2 = \frac{10}{10 + 26.667} \times 12$$
$$= 3.27273 \text{ A}$$

$\Rightarrow$ i_2 splits further

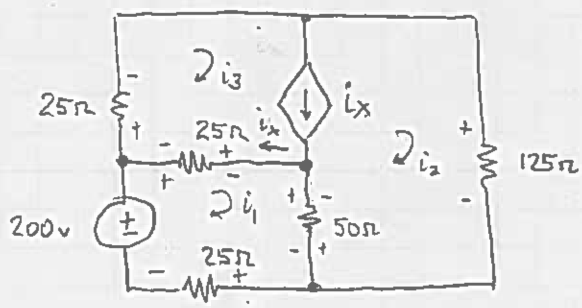


$$i_x = \frac{20}{20+10} \times i_2$$

$$\boxed{i_x = 2.18182 \text{ A}}$$

Question 2

(a) Intended method is to use mesh-current analysis



Mesh i_1:
$$-200 + 25(i_1 - i_3) + 50(i_1 - i_2) + 25 i_1 = 0$$
$$-200 + 25 i_1 - 25 i_3 + 50 i_1 - 50 i_2 + 25 i_1 = 0$$
$$100 i_1 - 50 i_2 - 25 i_3 = 200 \qquad (1)$$

Supermesh i_2, i_3:
$$25 i_3 + 125 i_2 + 50(i_2 - i_1) + 25(i_3 - i_1) = 0$$
$$25 i_3 + 125 i_2 + 50 i_2 - 50 i_1 + 25 i_3 - 25 i_1 = 0$$
$$-75 i_1 + 175 i_2 + 50 i_3 = 0 \qquad (2)$$

Supermesh dependence: $i_3 - i_2 = i_x \qquad (3)$

where $i_x = i_3 - i_1$

Substitute into (3): $i_3 - i_2 = i_3 - i_1$

giving $i_1 = i_2$ (4)

Now substitute (4) into (1) and (2)

$$100 i_1 - 50 i_1 - 25 i_3 = 200$$
$$50 i_1 - 25 i_3 = 200 \quad (5)$$

$$-75 i_1 + 175 i_1 + 50 i_3 = 0$$
$$100 i_1 + 50 i_3 = 0 \quad (6)$$

so $-50 i_3 = 100 i_1$

$i_3 = -2 i_1$

Substitute into (5): $50 i_1 - 25(-2 i_1) = 200$

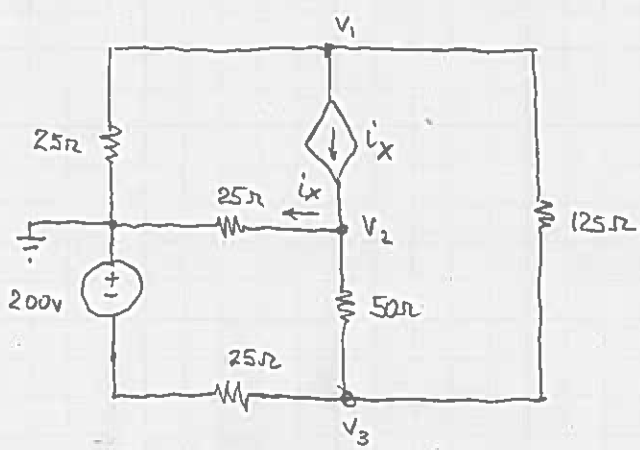
$100 i_1 = 200$

so $\boxed{i_1 = 2\text{ A}}$

and $\boxed{i_2 = 2\text{ A}}$

finally, $i_3 = -2 i_1$, giving $\boxed{i_3 = -4\text{ A}}$

Alternative method using node-voltage



The node equations ...

Node v_1: $\frac{V_1}{25} + i_x + \frac{V_1 - V_3}{125} = 0$

where the controlling current $i_x = \frac{V_2}{25}$.

so $\frac{V_1}{25} + \frac{V_2}{25} + \frac{V_1 - V_3}{125} = 0$

$(\times 125)$ $5V_1 + 5V_2 + V_1 - V_3 = 0$

$6V_1 + 5V_2 - V_3 = 0$ (1)

Node v_2: $\frac{V_2}{25} + \frac{V_2 - V_3}{50} - i_x = 0$

or $\frac{V_2}{25} + \frac{V_2 - V_3}{50} - \frac{V_2}{25} = 0$

$(\times 50)$ $2V_2 + V_2 - V_3 - 2V_2 = 0$

$V_2 = V_3$ (2)

Node v_3: $\frac{V_3 + 200}{25} + \frac{V_3 - V_2}{50} + \frac{V_3 - V_1}{125} = 0$

$(\times 250)$ $10V_3 + 2000 + 5V_3 - 5V_2 + 2V_3 - 2V_1 = 0$

$17V_3 - 5V_2 - 2V_1 = -2000$ (3)

Substitute (2) into (1) and (3)

$6V_1 + 5V_2 - V_2 = 0$

$6V_1 + 4V_2 = 0$ (4)

$17V_2 - 5V_2 - 2V_1 = -2000$

$12V_2 - 2V_1 = -2000$ (5)

From (5), $2V_1 = 12V_2 + 2000$, so $V_1 = 6V_2 + 1000$.
Substitute into (4)

$6(6V_2 + 1000) + 4V_2 = 0$

$36V_2 + 6000 + 4V_2 = 0$

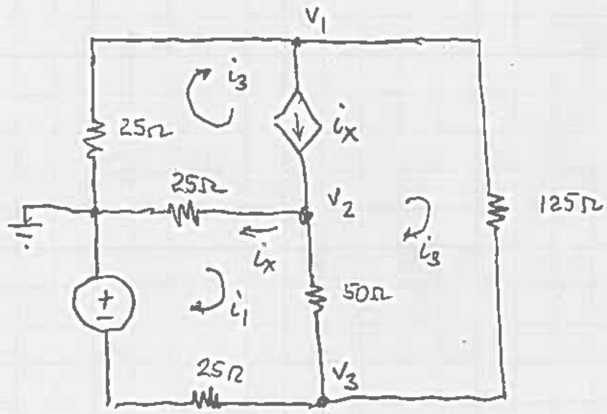
$40V_2 = -6000$

so $V_2 = -150$ V

Therefore, $V_1 = 6V_2 + 1000 = 6(-150) + 1000 = 100$ V.

And $V_3 = V_2$, so $V_3 = -150$ V.

Now the mesh currents:



Looking at mesh 3: $i_3 = -\frac{V_1}{25} = -\frac{100}{25}$

$$\boxed{i_3 = -4A}$$

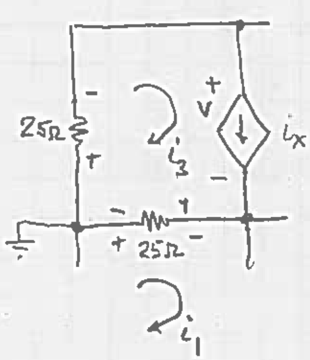
In mesh 1: $i_1 = \frac{V_3 - (-200)}{25} = \frac{-150 + 200}{25}$

$$\boxed{i_1 = 2A}$$

In mesh 3: $i_3 = \frac{V_1 - V_3}{125} = \frac{100 - (-150)}{125}$

$$\boxed{i_2 = 2A}$$

(b) Intended method by mesh-current analysis



Mesh i_3: $25 i_3 + V + 25(i_3 - i_1) = 0$

so $25(-4) + V + 25(-4-2) = 0$

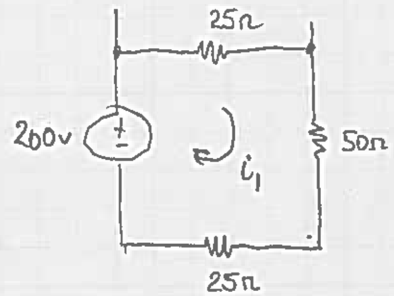
$-100 + V - 150 = 0$

$V = 250V$

Power: $P_i = v i_x = 250(i_3 - i_1)$

$= 250(-6)$

$$\boxed{P_i = -1500W}$$



$$p_1 = -v i_1 = -200(2)$$

$$\boxed{p_i = -400 \text{ W}}$$

(c) Node V_1:

$$\frac{V_1}{25} + i_x + \frac{V_1 - V_3}{125} = 0$$

where $i_x = V_2/25$

so

$$\frac{V_1}{25} + \frac{V_2}{25} + \frac{V_1 - V_3}{125} = 0$$

Node V_2:

$$\frac{V_2}{25} + \frac{V_2 - V_3}{50} - \frac{V_2}{25} = 0$$

i_x

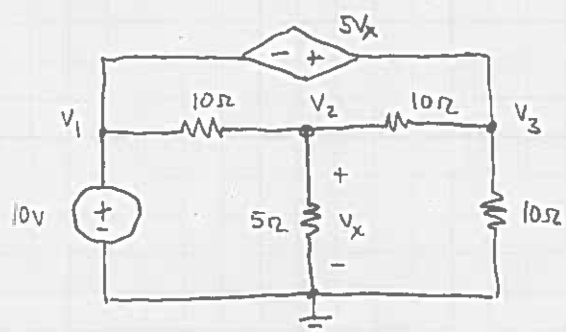
[simplifies to $V_2 = V_3$]

Node V_3:

$$\frac{V_3 + 200}{25} + \frac{V_3 - V_2}{50} + \frac{V_3 - V_1}{125} = 0$$

Question 3

(a) Using the node-voltage method



With the bottom of the circuit as reference, we see

$$V_1 = 10\text{ V}$$

AND

$$V_3 = V_1 + 5V_x = 10 + 5V_x$$

This leaves a single node V_2 unknown

Node V_2 : $$\frac{V_2 - 10}{10} + \frac{V_2 - V_3}{10} + \frac{V_2}{5} = 0$$

$$(\times 10) \quad V_2 - 10 + V_2 - V_3 + 2V_2 = 0$$
$$4V_2 - V_3 = 10$$

Substituting for V_3 : $$4V_2 - (10 + 5V_x) = 10$$

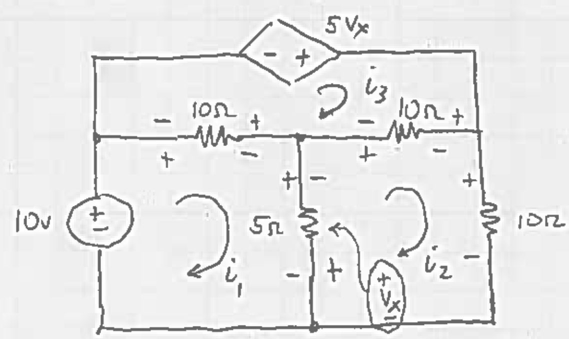
where we note $V_x = V_2$.

$$4V_2 - 10 - 5V_2 = 10$$
$$-V_2 = 20$$

$$V_2 = -20$$

Therefore, $\boxed{V_x = -20}$

Using the mesh-current method,



Mesh i_1 : $$-10 + 10(i_1 - i_3) + 5(i_1 - i_2) = 0$$
$$15 i_1 - 5 i_2 - 10 i_3 = 10 \qquad (1)$$

Mesh i_2 : $$5(i_2 - i_1) + 10(i_2 - i_3) + 10 i_2 = 0$$
$$-5 i_1 + 25 i_2 - 10 i_3 = 0 \qquad (2)$$

Mesh i_3 : $$10(i_3 - i_1) - 5V_x + 10(i_3 - i_2) = 0$$
$$-10 i_1 - 10 i_2 + 20 i_3 = 5V_x \qquad (3)$$

We also know that $V_x = 5(i_1 - i_2)$, so equation (3) becomes

$$-10i_1 - 10i_2 + 20i_3 = 25i_1 - 25i_2$$
$$-35i_1 + 15i_2 + 20i_3 = 0 \qquad (4)$$

Add equation (4) to 3 × equation (1)

$$\begin{array}{rl} -35i_1 + 15i_2 + 20i_3 = 0 & (4) \\ 45i_1 - 15i_2 - 30i_3 = 30 & (1)\times 3 \\ \hline 10i_1 - 10i_3 = 90 & \end{array}$$

so $i_1 = 3 + i_3$ and $i_3 = i_1 - 3$

Substitute into (2) and (4)

$$-5i_1 + 25i_2 - 10(i_1 - 3) = 0$$
$$-15i_1 + 25i_2 = -30 \qquad (5)$$

$$-35i_1 + 15i_2 + 20(i_1 - 3) = 0$$
$$-15i_1 + 15i_2 = 60 \qquad (6)$$

Add −1 × equation (5) to (6)

$$\begin{array}{r} 15i_1 - 25i_2 = 30 \\ -15i_1 + 15i_2 = 60 \\ \hline -10i_2 = 90 \end{array}$$

so $i_2 = -9$ A

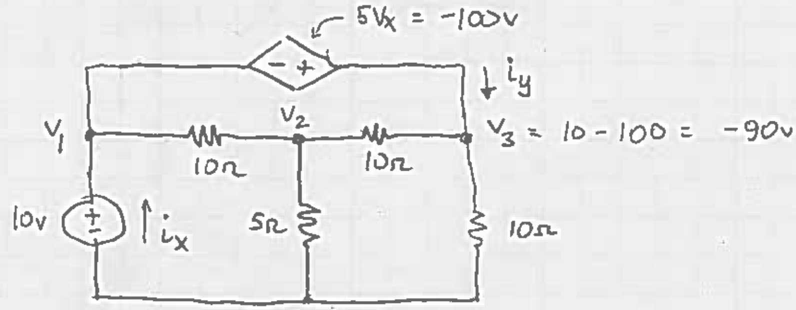
Substitute back into (6)

$$-15i_1 + 15(-9) = 60$$
$$-15i_1 = 60 + 135$$
$$i_1 = -13 \text{ A}$$

We don't need i_3, and then $V_x = 5(i_1 - i_2)$
$= 5(-13 + 9)$

$$\boxed{V_x = -20\text{v}}$$

(b) From the node-voltage method



Need to KCL at V_1, but first need i_y. at node V_3

$$\frac{V_3 - V_2}{10} + \frac{V_3}{10} - i_1 = 0$$

$$\frac{-90-(-20)}{10} + \frac{-90}{10} = i_y$$

$$-7 - 9 = i_y \quad , \quad \text{so} \quad i_y = -16\ A$$

Then, at node V_1

$$-i_x + i_y + \frac{V_1 - V_2}{10} = 0$$

$$i_x = -16 + \frac{10-(-20)}{10}$$

$$\boxed{i_x = -13\ A}$$

Using the mesh-current method, from our previous calculation,

$$\boxed{i_x = i_1 = -13\ A}$$