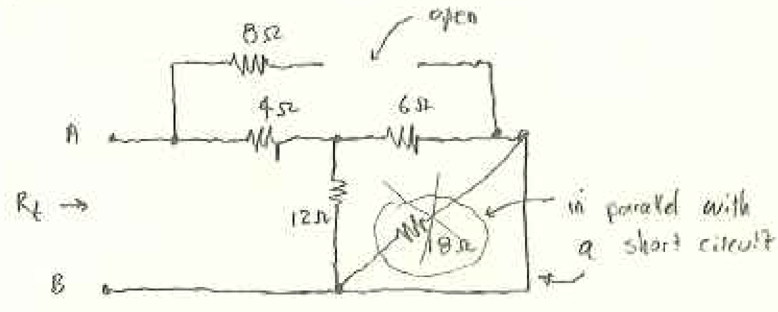
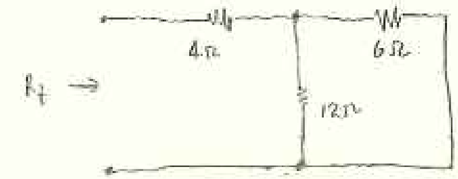
Question 1

(a) Since there are no dependent sources, we may zero the independent ones.

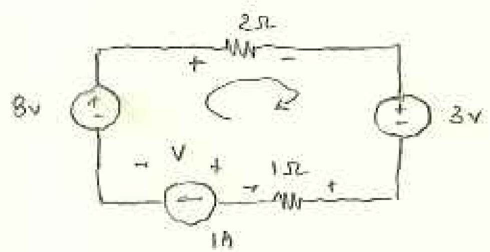


or redrawing



$$R_t = (12 \parallel 6) + 4 = \boxed{8\Omega}$$

(b)



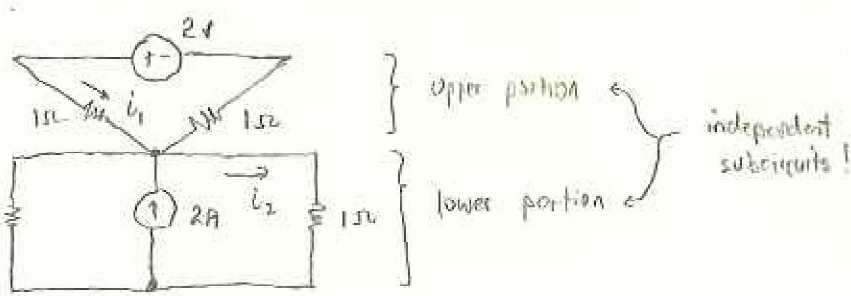
Need to find voltage v. Common loop current $i = 1A$. KVL around the loop gives

$$-8 + 2i + 3 + 1i + V = 0$$
$$-8 + 2 + 3 + 1 + V = 0$$
$$V = 2v$$

Power $p_{1A} = iV = 1A \times 2V = \boxed{2\,W}$

This is positive, so power is <u>absorbed</u>.

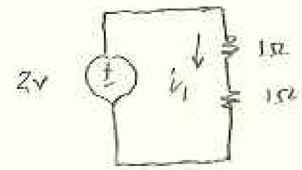
(c)



Note that the upper portion of the circuit is independent of the lower part, since there is no return path for current between the two portions.

The upper part redrawn

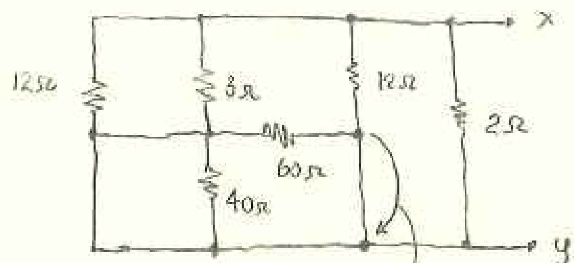
$$i_1 = \frac{V}{R} = \frac{2}{1+1} = \boxed{1\text{ A}}$$



The lower part is a simple current divider

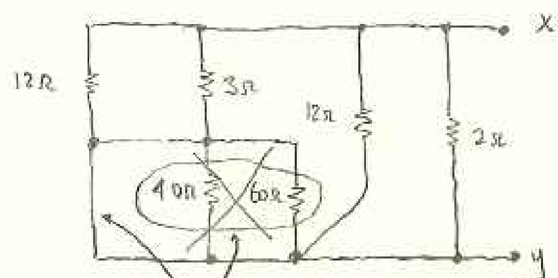
$$i_2 = \frac{1\Omega}{1\Omega + 1\Omega} \times 2\text{ A} = \boxed{1\text{ A}}$$

(d)



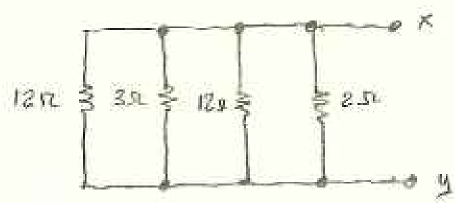
straightening resistor branches.

drag this node down to the bottom wire.



in parallel with a short circuit

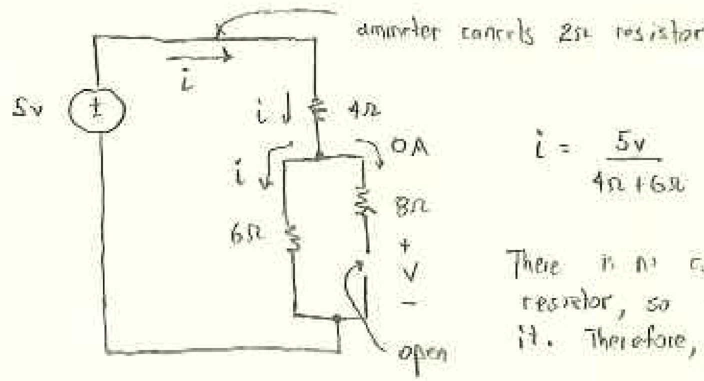
⇓



$$R_{eq} = \left(\frac{1}{12} + \frac{1}{3} + \frac{1}{12} + \frac{1}{2}\right)^{-1}$$

$$= \boxed{1\,\Omega}$$

(e) An equivalent circuit using the internal resistances of the voltmeter and ammeter



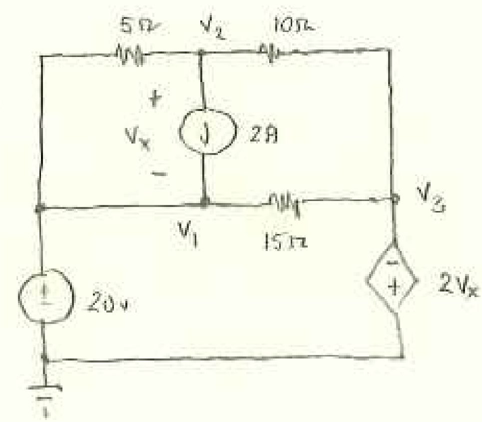
$$i = \frac{5\,\text{v}}{4\Omega + 6\Omega} = \boxed{0.5\text{ A}}$$

There is no current in the 8Ω resistor, so no voltage across it. Therefore,

$$v = i \times 6\Omega = \boxed{3\text{V}}$$

Question 2

(a)



We see that $V_1 = 20$ v, and also $V_3 = -2V_x$.

Node V_2: $\quad \dfrac{V_2 - 20}{5} + 2 + \dfrac{V_2 - V_3}{10} = 0$

$(\times 10)\quad 2V_2 - 40 + 20 + V_2 - V_3 = 0$

$3V_2 - V_3 = 20$

Substituting for V_3

$$3V_2 - (-2V_x) = 20$$
$$3V_2 + 2V_x = 20$$

We also know that $V_x = V_2 - V_1$, so

$$3V_2 + 2(V_2 - V_1) = 20$$
$$3V_2 + 2V_2 - 40 = 20$$
$$5V_2 = 60$$

so $V_2 = 12$

Also, $V_x = V_2 - 20 = -8$

and $V_3 = -2V_x = 16$

$$\boxed{\begin{aligned} V_2 &= 12 \\ V_3 &= 16 \end{aligned}}$$

(b) Meshes i_1 and i_2 form a supermesh

Supermesh i_1, i_2:

$$5i_1 + 10i_2 + 15(i_2 - i_3) = 0$$
$$5i_1 + 10i_2 + 15i_2 - 15i_3 = 0$$
$$5i_1 + 25i_2 - 15i_3 = 0 \qquad (1)$$

Supermesh dependence: $i_1 - i_2 = 2 \qquad (2)$

Mesh i_3:

$$-20 + 15(i_3 - i_2) - 2V_x = 0$$
$$15i_3 - 15i_2 - 2V_x = 20 \qquad (3)$$

We need to express V_x in terms of mesh currents.
In mesh i_1, we have

$$5i_1 + V_x = 0, \quad \text{so} \quad V_x = -5i_1$$

Substitute into (3)

$$15i_3 - 15i_2 + 10i_1 = 20 \qquad (4)$$

If we add equations (1) and (4), we can eliminate i_3

$$\begin{array}{rrrl} 5i_1 & + 25i_2 & - 15i_3 & = 0 \\ 10i_1 & - 15i_2 & + 15i_3 & = 20 \\ \hline 15i_1 & + 10i_2 & & = 20 \end{array} \qquad (5)$$

From (2), $i_1 = 2 + i_2$, so equation (5) becomes

$$15(2 + i_2) + 10i_2 = 20$$
$$30 + 15i_2 + 10i_2 = 20$$
$$25i_2 = -10$$

$$\text{so} \quad i_2 = -0.4$$

From (2), $i_1 = 1.6\text{ A}$, and from (1), $i_3 = -0.1333\text{ A}$

$$\boxed{\begin{aligned} i_1 &= 1.6\text{ A} \\ i_2 &= -0.4\text{ A} \\ i_3 &= -0.1333\text{ A} \end{aligned}}$$

(c) Using $i_3 = -0.1333$ A from part (b)

$$P_1 = -vi_3 = -20 \times -0.1333 = \boxed{\begin{array}{c}2.6667\text{ W}\\ \text{(absorbed)}\end{array}}$$

Using i_3 again from part (b), we also need V_x.

From part (a), $V_x = V_2 - V_1 = 12 - 20 = -8$ V
From part (b), $V_x = -5i_1 = -5(1.6) = -8$ V

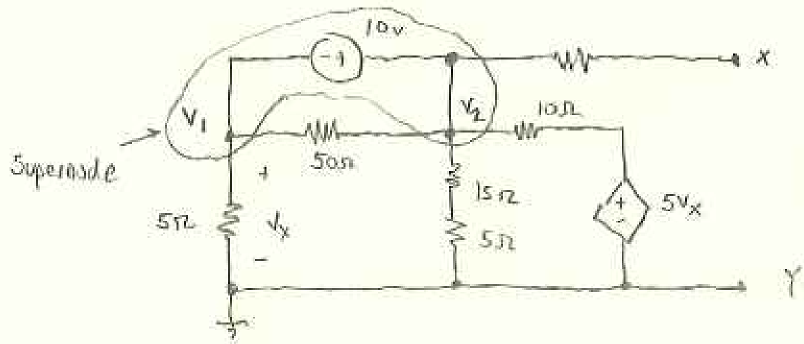
$$\text{so} \quad P_2 = -(2V_x)i_3 = 16i_3 = \boxed{\begin{array}{c}-2.333\text{ W}\\ \text{(delivered)}\end{array}}$$

Using V_x from either part (a) or (b)

$$P_3 = iV_x = 2 \times -8 = \boxed{\begin{array}{c}-16\text{W}\\ \text{delivered}\end{array}}$$

Question 3

(a) Thevenin voltage.



$$\text{Supernode } V_1, V_2: \quad \frac{V_1}{5} + \cancel{\frac{V_1 - V_2}{50}} + \cancel{\frac{V_2 - V_1}{50}} + \frac{V_2}{15+5} + \frac{V_2 - 5V_x}{10} = 0$$

$$(\times 20) \quad 4V_1 + V_2 + 2V_2 - 10V_x = 0$$

$$4V_1 + 3V_2 - 10V_x = 0 \qquad (1)$$

$$\text{Dependence:} \quad V_2 - V_1 = 10 \qquad (2)$$

We also see that $V_x = V_1$, so equation (1) becomes

$$4V_1 + 3V_2 - 10V_1 = 0$$

$$-6V_1 = -3V_2, \quad \text{so} \quad V_2 = 2V_1 \text{ or } V_1 = \tfrac{1}{2}V_2$$

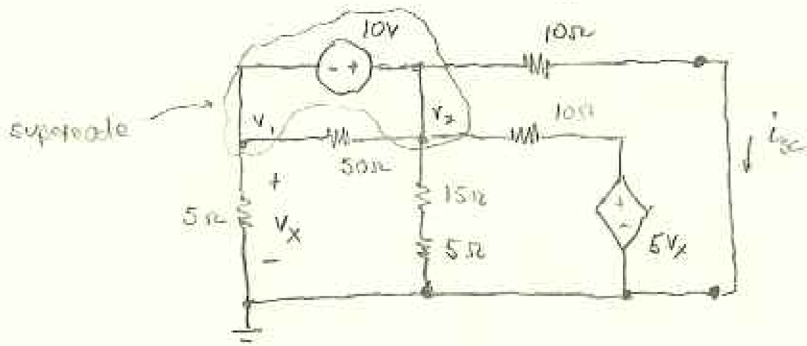
From (2), $V_2 - \frac{1}{2}V_2 = 10$

so $V_2 = 20$ V

Since the terminal X is open-circuited, no current flows in the 10Ω resistor connecting V_2 and X, so

$$\boxed{V_t = V_X = V_2 = 20V}$$

Thevenin resistance: Since there is a dependent source, we need to find i_{sc}.



Supernode V_1, V_2: $\frac{V_1}{5} + \cancel{\frac{V_1 - V_2}{50}} + \cancel{\frac{V_2 - V_1}{50}} + \frac{V_2}{20} + \frac{V_2 - 5V_X}{10} + \frac{V_2}{10} = 0$

This is i_{sc}.

(×20) $4V_1 + V_2 + 2V_2 - 10V_X + 2V_2 = 0$

$4V_1 + 5V_2 - 10V_X = 0$

As before, $V_X = V_1$, so $4V_1 + 5V_2 - 10V_1 = 0$

$-6V_1 + 5V_2 = 0$ (1)

The dependence equation, also as before, $V_2 - V_1 = 10$ (2)

From (2), $V_1 = V_2 - 10$, so from (1)

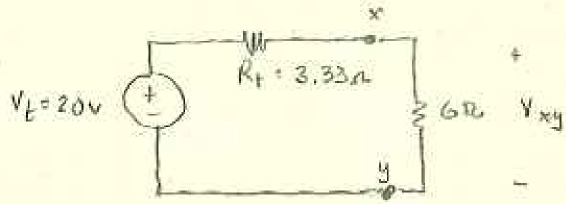
$$-6(V_2 - 10) + 5V_2 = 0$$
$$-6V_2 + 60 + 5V_2 = 0$$

$$V_2 = 60 \text{ V}$$

The short-circuit current $i_{sc} = \frac{V_2}{10} = 6A$

Therefore, $R_t = \frac{V_t}{i_{sc}} = \frac{20}{6} = \boxed{3.333\Omega}$

(b) Using the Thevenin equivalent,



By voltage division, $V_{xy} = \frac{6\Omega}{6\Omega + 3.33\Omega} \times 20v = \boxed{12.86v}$