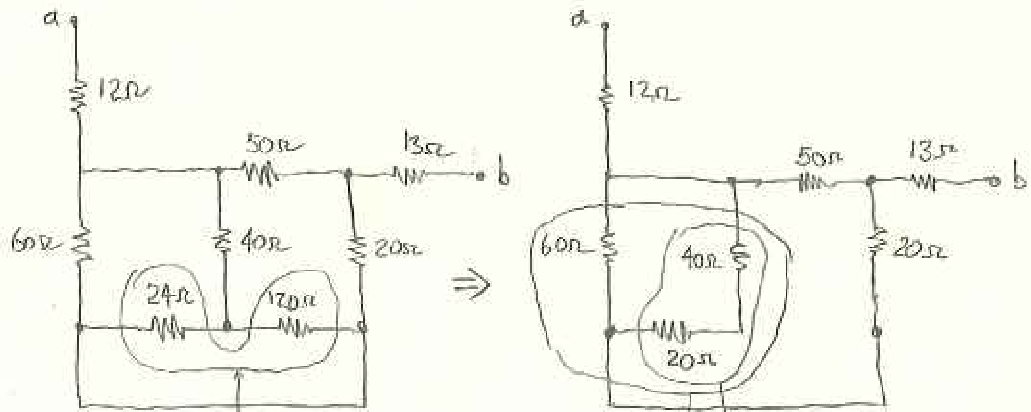
Question 1

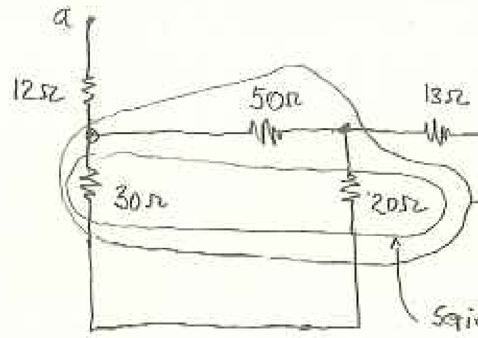
(a)



in parallel $24 // 120 = 20\Omega$

series 60Ω

parallel $60 // 60 = 30\Omega$

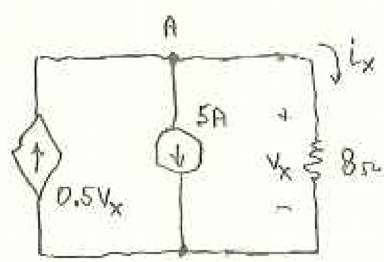


parallel $50 // 50 = 25\Omega$

Series 50Ω

Series $R_{ab} = 12 + 25 + 13$
$= \underline{\underline{50\Omega}}$

(b)



KCL at A gives

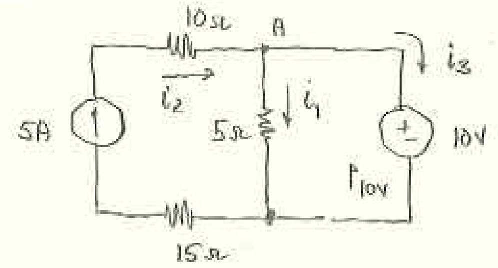
$$0.5V_x - 5 - i_x = 0$$

and $V_x = 8i_x$, or $i_x = V_x/8$

Therefore,
$$0.5V_x - 5 - V_x/8 = 0$$
$$0.5V_x - 5 - 0.125V_x = 0$$

$0.375V_x = 5$, so $V_x = \underline{\underline{13.333\text{ V}}}$

(c)



With the current directions defined as shown, $i_2 = 5A$, and

$$i_1 = \frac{10V}{5\Omega} = 2A.$$

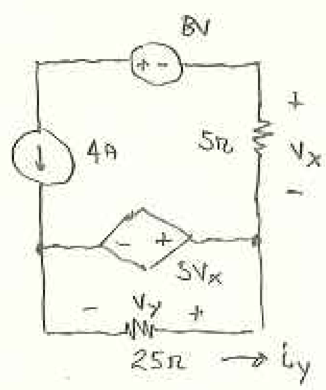
KCL at node A gives $i_2 - i_1 - i_3 = 0$, so $i_3 = i_2 - i_1$

$$i_3 = 5 - 2 = 3A$$

The power in the 10V source $P_{10V} = +10V \times i_3$
$= +30W$

$|P_{10V}| = \underline{\underline{30W}}$, and this is <u>absorbed</u>

(d)



The voltage across the 25Ω resistor is indicated here as V_y.

V_y <u>must</u> be $V_y = 5V_x$ (parallel)

so $i_y = -\frac{V_y}{25} = -\frac{5V_x}{25}$

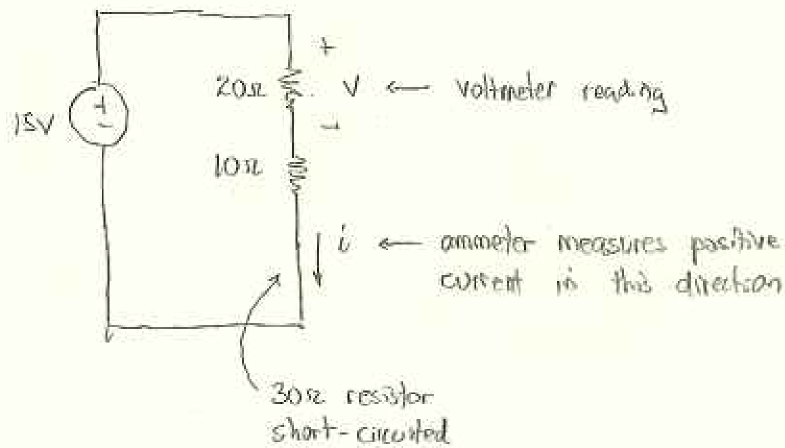
$= -V_x/5$

The current through the 5Ω resistor <u>must</u> be 4A in the direction of the current source (series).

so $V_x = -4A \times 5\Omega$
$= -20V$

Therefore, $i_y = -V_x/5$
$= -(-20)/5$
$= \underline{\underline{4A}}$

(e)

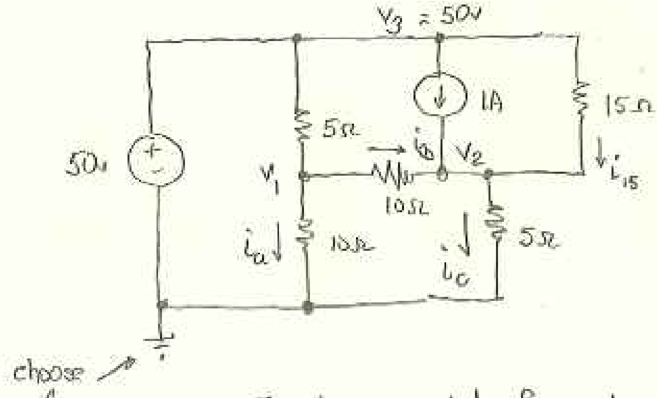


We have a simple voltage divider: $V = \frac{20}{20+10} \times 15$

$V = \underline{\underline{10\,V}}$

And $i = \frac{15}{20+10} = \underline{\underline{0.5\,A}}$

Question 2



Equations needed for nodes V_1 and V_2

Node V_1: $\frac{V_1}{10} + \frac{V_1 - 50}{5} + \frac{V_1 - V_2}{10} = 0$

$(\times 10)$ $V_1 + 2V_1 - 100 + V_1 - V_2 = 0$

$4V_1 - V_2 = 100$ (1)

Node V_2: $\frac{V_2 - V_1}{10} - 1 + \frac{V_2}{5} + \frac{V_2 - 50}{15} = 0$

$(\times 30)$ $3V_2 - 3V_1 - 30 + 6V_2 + 2V_2 - 100 = 0$

$11V_2 - 3V_1 = 130$ (2)

From (1), $V_2 = 4V_1 - 100$. Substitute into (2)

$11(4V_1 - 100) - 3V_1 = 130$

$44V_1 - 1100 - 3V_1 = 130$

$41V_1 = 1230$, so $\underline{V_1 = 30v}$.

From (2), $V_2 = 4(30) - 100$, so $\underline{V_2 = 20v}$.

current i_a: $i_a = \frac{V_1}{10\Omega} = \underline{\underline{3A}}$

current i_b: $i_b = \frac{V_1 - V_2}{10\Omega} = \frac{30-20}{10} = \underline{\underline{1A}}$

current i_c: $i_c = \frac{V_2}{5\Omega} = \frac{20}{5} = \underline{\underline{4A}}$

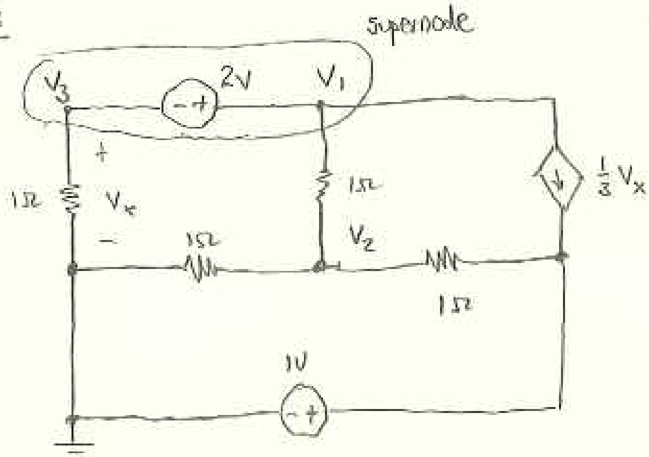
With the current i_{15} in the 15Ω resistor as labeled,

$$i_{15} = \frac{50 - V_2}{15\Omega} = \frac{50-20}{15} = 2A$$

The power $P_{15} = i_{15}^2 \times 15\Omega$
$= 2^2 \times 15$

$$P_{15} = \underline{\underline{60W}}$$

Question 3



(a) Node-voltage method.

Supernode: $\frac{V_3}{1} + \frac{V_1 - V_2}{1} + \frac{1}{3}V_x = 0$

$$V_3 + V_1 - V_2 + \frac{1}{3}V_x = 0$$

Note that $V_x = V_3$ in the above circuit

$$\frac{4}{3}V_3 + V_1 - V_2 = 0 \qquad (1)$$

Supernode dependence: $V_1 - V_3 = 2V$

or $V_3 = V_1 - 2$

Substitute into (1) giving $\frac{4}{3}(V_1 - 2) + V_1 - V_2 = 0$

$$\frac{7}{3}V_1 - V_2 = \frac{8}{3}$$

$$(\times 3) \qquad 7V_1 - 3V_2 = 8 \qquad (2)$$

Node V_2: $\frac{V_2}{1} + \frac{V_2 - 1}{1} + \frac{V_2 - V_1}{1} = 0$

$$V_2 + V_2 - 1 + V_2 - V_1 = 0$$

$$3V_2 - V_1 = 1 \qquad (3)$$

From (3), $V_1 = 3V_2 - 1$. Substitute into (2)

$$7(3V_2 - 1) - 3V_2 = 8$$
$$21V_2 - 7 - 3V_2 = 8$$
$$18V_2 = 15$$

so $V_2 = \frac{15}{18} = \underline{\underline{\frac{5}{6}\text{ V}}}$

Substitute back into (3): $3\left(\frac{5}{6}\right) - V_1 = 1$

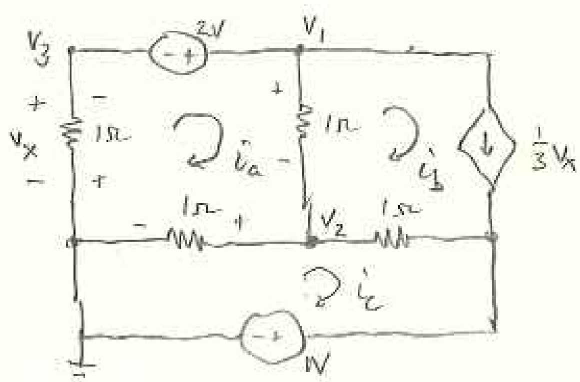
$$V_1 = \frac{15}{6} - 1 = \frac{9}{6}$$

$$\underline{\underline{V_1 = \frac{3}{2}\text{ V}}}$$

Finally, from the supernode dependence $V_3 = V_1 - 2$

$$V_3 = \frac{3}{2} - 2 = \underline{\underline{-\frac{1}{2}\text{ V}}}$$

(b) Mesh-current method



mesh a: $-2 + 1 \times (i_a - i_b) + 1 \times (i_a - i_c) + i_a \times 1 = 0$

$$-2 + i_a - i_b + i_a - i_c + i_a = 0$$

$$3i_a - i_b - i_c = 2 \qquad (1)$$

mesh c: $1 + 1\times(i_c - i_a) + 1\times(i_c - i_b) = 0$

$$1 + i_c - i_a + i_c - i_b = 0$$

$$2i_c - i_a - i_b = -1 \qquad (2)$$

mesh b: $i_b = \frac{1}{3}V_x$, where $V_x = -1\times i_a$

Therefore, $i_b = -\frac{1}{3}i_a \qquad (3)$

Substitute (3) into (1) and (2)

$$(1) \rightarrow \quad 3i_a + \frac{1}{3}i_a - i_c = 2$$

$$\frac{10}{3}i_a - i_c = 2$$

$$(\times 3) \quad 10i_a - 3i_c = 6 \qquad (4)$$

$$(2) \rightarrow \quad 2i_c - i_a + \frac{1}{3}i_a = -1$$

$$2i_c - \frac{2}{3}i_a = -1$$

$$(\times 3) \quad 6i_c - 2i_a = -3 \qquad (5)$$

From (4), $i_c = \frac{10}{3}i_a - 2$. Substitute into (5)

$$6\left(\frac{10}{3}i_a - 2\right) - 2i_a = -3$$

$$20i_a - 12 - 2i_a = -3$$

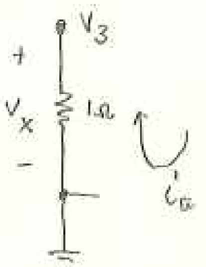
$$18i_a = 9$$

$$\underline{i_a = \tfrac{1}{2}\,A}$$

And from (4), $i_c = \left(\frac{10}{3}\right)\left(\frac{1}{2}\right) - 2 = \underline{-\frac{1}{3}\,A}$

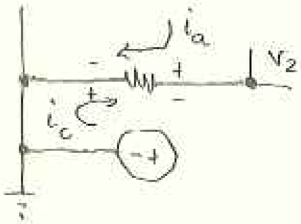
And from (3), $i_b = \left(-\frac{1}{3}\right)\left(\frac{1}{2}\right) = \underline{-\frac{1}{6}\,A}$

From the circuit, we see that



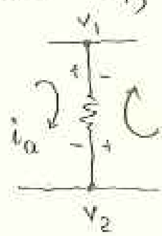
$V_3 = V_x = -\frac{1}{2}\times 1 = \underline{\underline{-\frac{1}{2}\,V}}$

For node V_2,



$$V_2 = (i_a - i_c) \times 1\Omega$$
$$= \frac{1}{2} - \left(-\frac{1}{3}\right)$$
$$= \underline{\underline{\frac{5}{6}\text{ V}}}$$

For node V_1,



$$V_1 = V_2 + (i_a - i_b) \times 1\Omega$$
$$= \frac{5}{6} + \frac{1}{2} - \left(\frac{1}{6}\right)$$
$$= \underline{\underline{\frac{3}{2}\text{ V}}}$$