Last Name: Solutions

First Name: __________

Lecture Section: ______

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ENGG 225 - Fundamentals of Electrical Circuits and Machines

Midterm Examination

Thursday, February 27, 2020
Time: 7:00 - 9:00 PM

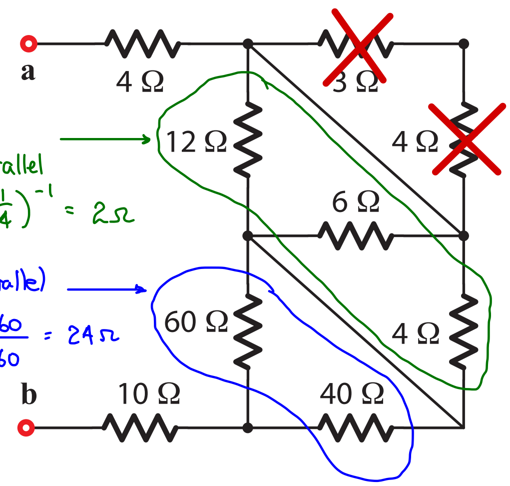
Instructions:

- Time allowed is 2 hours.
- The examination is closed-book.
- Only calculators sanctioned by the Schulich School of Engineering (Casio FX-260, Casio FX-300MS, or TI-30XIIS) are permitted in the examination.
- The maximum number of marks is 40, as indicated; please attempt all questions. The midterm examination counts 25% toward the final grade.
- Please use a pen or heavy pencil to ensure legibility.
- Please answer questions in the spaces provided; if space is insufficient, please use the back of the pages.
- Please show your work; where appropriate, marks will be awarded for proper and well-reasoned explanations.

UCID: ____________________

1. **[15 marks.]** Parts (a)-(e) below each have an identical weighting of three marks. Please answer the questions in the boxes provided.

(a) **[3]** In the circuit at right, find the total equivalent resistance R_{ab} between the terminals **a** and **b**.



parallel

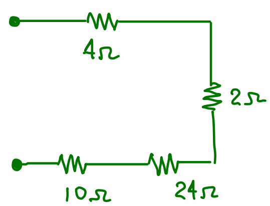
$\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right)^{-1} = 2\Omega$

paralle)

$\frac{40\times 60}{40+60} = 24\Omega$

Note 3Ω and 4Ω resistors in series, and then in parallel with a wire.

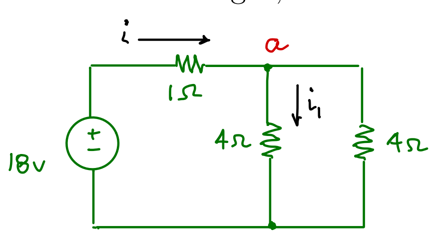
Circuit simplifies to:

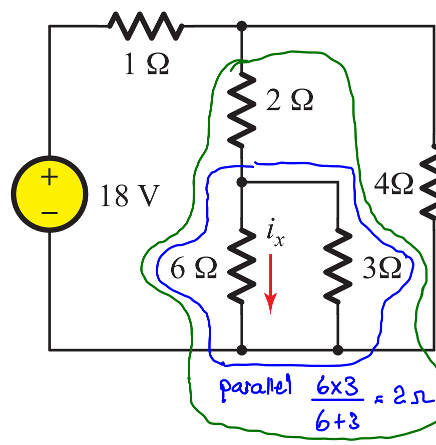


$R_{ab} = 4+2+24+10$
$= 40\Omega$

Answer: $R_{ab} =$ 40Ω

(b) **[3]** In the circuit at right, find the current i_x.





parallel $\frac{6\times 3}{6+3} = 2\Omega$

series $2+2 = 4\Omega$

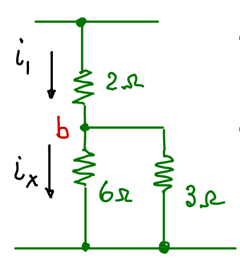
Total equivalent resistance across the source:

$R_{eq} = 4/\!/4 + 1$
$= 3\Omega$

Therefore, $i = 18v/3\Omega = 6\,A$

Current division at node a:

$i_1 = \frac{4}{4+4} \times i$
$= 3\,A$

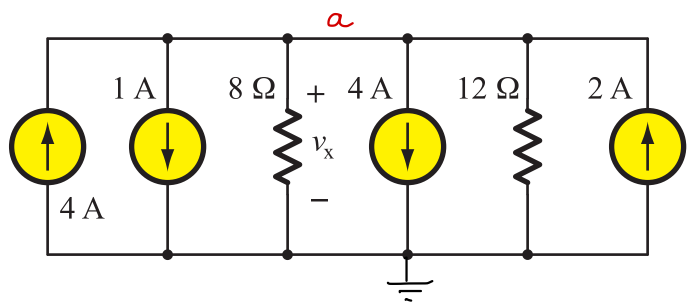


Answer: $i_x =$ 1 A

Current division at node b:

$i_x = \frac{3}{6+3} \times i_1$
$= 1A$

(c) **[3]** Calculate the voltage v_x in the circuit at right.



Many ways to solve this.
Node a is the whole top wire.

Node equation:

Node a: $-4 + 1 + \frac{V_a}{8} + 4 + \frac{V_a}{12} - 2 = 0$

$(\times 24)\quad -96 + 24 + 3V_a + 96 + 2V_a - 48 = 0$

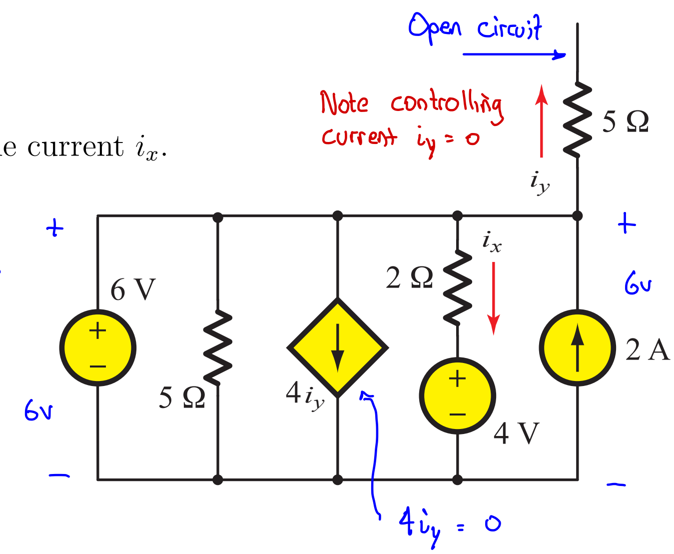
$5V_a = 24$

so $V_a = 4.8$.

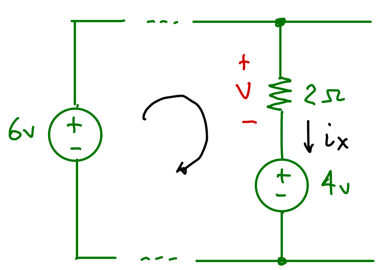
And $V_a = V_x$.

Answer: $v_x =$ 4.8v

(d) **[3]** In the circuit at right, calculate the current i_x.



Note 5 parallel branches with 6v across each.



KVL around this loop gives

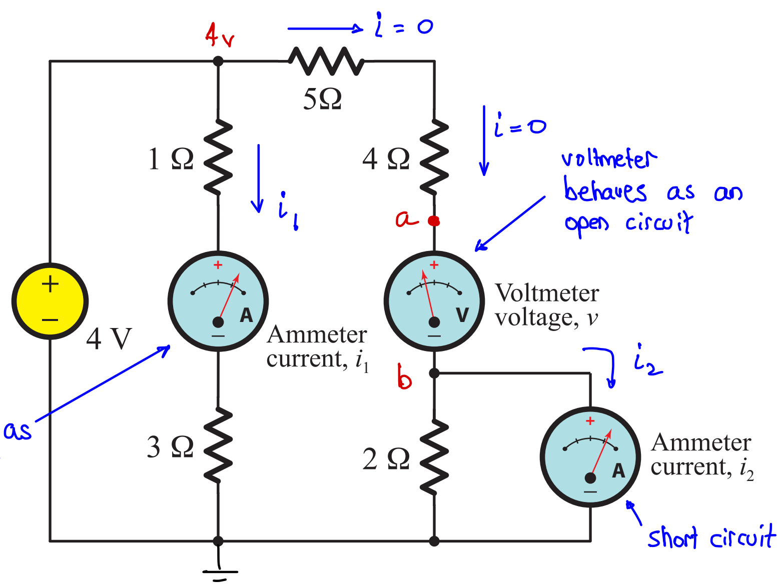
$-6 + V + 4 = 0$

$V = 2v$

Therefore, $i_x = \frac{2v}{2\Omega} = 1A$

Answer: $i_x =$ 1 A

(e) [**3**] In the circuit at right, assume that the voltmeter and ammeters are ideal. Give the voltmeter's reading v, and the two ammeter readings i_1 and i_2.



Current i_1

$$i_1 = \frac{4v}{(1+3)\Omega} = 1A$$

Current i_2

There is no path for current through voltmeter, so $i = 0$ and $i_2 = 0$

Voltage v

Node b: $V_b = 0$ (ammeter is short-circuit to ground)

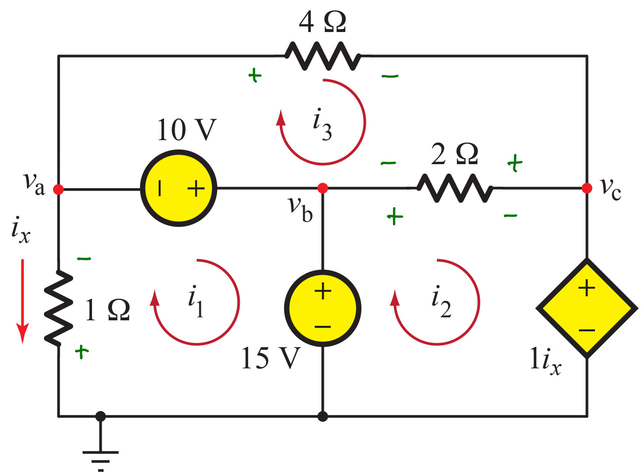
Node a: $V_a = 4v$ (same as voltage across the source because $i = 0$ through 5Ω and 4Ω resistors)

Answer: $v =$ 4v

Answer: $i_1 =$ 1A

Answer: $i_2 =$ 0 A

2. **[11 marks.]** Consider the circuit below.



(a) **[8]** Determine the mesh currents i_1, i_2, i_3.

(b) **[3]** Using your answers to part (a), calculate the node voltages v_a, v_b, v_c.

Mesh currents *(Amperes)*

Parameter	Value
i_1	−5A
i_2	5A
i_3	0A

Node voltages *(Volts)*

Parameter	Value
v_a	5v
v_b	15v
v_c	5v

Mesh-current equations

Mesh i_1: $15 + 1 \times i_1 - 10 = 0$

so $\boxed{i_1 = -5A}$

Mesh i_2: $-15 + 2(i_2 - i_3) + 1 i_x = 0$

$2i_2 - 2i_3 + i_x = 15$ —— (1)

Mesh i_3: $10 + 4i_3 + 2(i_3 - i_2) = 0$

$-2i_2 + 6i_3 = -10$ —— (2)

For the dependent source, we see that

$i_x = -i_1 = 5A$

Substitute into equation (1)

$2i_2 - 2i_3 + 5 = 15$

$2i_2 - 2i_3 = 10$ —— (3)

Solve for i_2 and i_3. Add equations (2) and (3):

$$\begin{array}{r} -2i_2 + 6i_3 = -10 \\ 2i_2 - 2i_3 = 10 \\ \hline 4i_3 = 0, \end{array}$$

so $\boxed{i_3 = 0}$

Then, from equation (3), $2i_2 - 0 = 10$, so $\boxed{i_2 = 5A}$

(Question 2, additional workspace ...)

By inspection at nodes b and c:

Node b: $\boxed{V_b = 15v}$

Node c: $V_c = 1 \cdot i_x = -i_c$ $\boxed{V_c = 5v}$

Two ways to solve for V_a.

1. KVL around mesh i_1

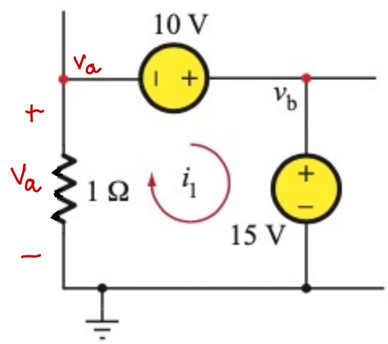
$-V_a - 10 + 15 = 0$

$\boxed{V_a = 5v.}$

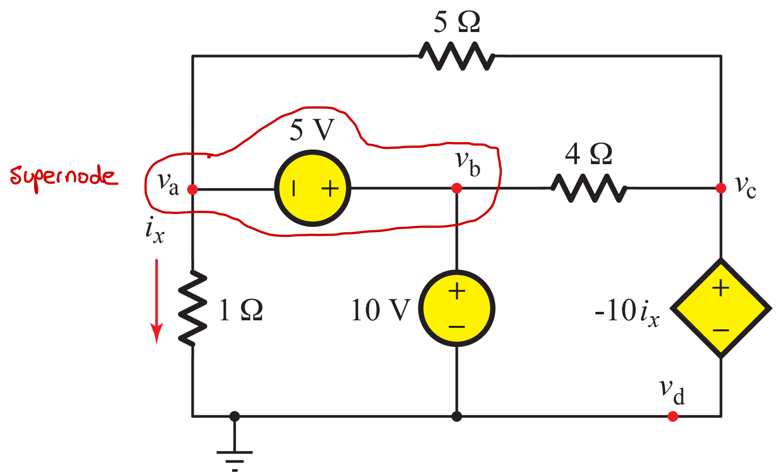
2. We know $i_1 = -5A$, so

$V_a = -1 \times i_a$

$\boxed{V_a = 5v.}$



3. **[14 marks.]** Consider the circuit below.



(a) **[8]** Use the node-voltage method to solve for the node voltages v_a, v_b, v_c.

(b) **[6]** Using your answers to part (a), calculate the power in all the circuit elements, and confirm energy balance.

Node voltages *(Volts)*

Parameter	Value
v_a	5v
v_b	10v
v_c	−50v
v_d	0v (reference)

Power *(Watts)*

Parameter	Value
$p_{1\Omega}$	25 W
$p_{4\Omega}$	605 W
$p_{5\Omega}$	900 W
p_{10V}	−310 W
p_{5V}	80 W
p_{-10ix}	−1300 W
Total	0 W

We immediately know that

$$\boxed{V_b = 10v}$$

Nodes a and b form a supernode, for which the dependence equation is

$$V_b - V_a = 5v$$

With $V_b = 10v$, we will therefore know V_a immediately:

$$V_a = V_b - 5 = 10 - 5$$

$$\boxed{V_a = 5v}$$

Finally, for node c

$$V_c = -10\, i_x \qquad \text{——— (1)}$$

where we see that

$$i_x = \frac{V_a}{1\Omega} = 5\text{ A}$$

Therefore, from (1),

$$V_c = -10 \times 5 = -50v$$

$$\boxed{V_c = -50v}$$

(Question 3, additional workspace ...)

Power calculations

1Ω resistor

$$P_{1\Omega} = i_x^2 R = (5)^2 \times 1 = 25\,W$$

5Ω resistor

$$P_{5\Omega} = \frac{(V_a - V_c)^2}{5} = \frac{(V_c - V_a)^2}{5} = \frac{(-50-5)^2}{5} = 605\,W$$

4Ω resistor

$$P_{4\Omega} = \frac{(V_b - V_c)^2}{4} = \frac{(V_c - V_b)^2}{4} = \frac{(-50-10)^2}{4} = 900\,W$$

The 5-volt source

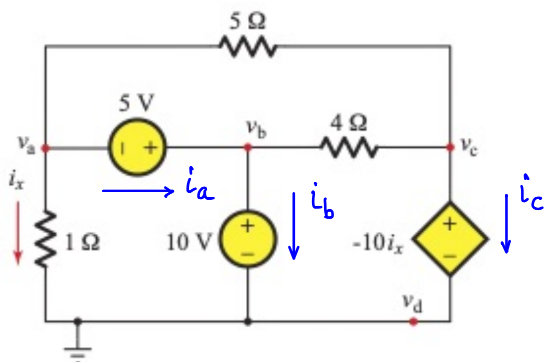
We will need a KCL calculation at node a to find i_a

$$\frac{V_a}{1} + \frac{V_a - V_c}{5} + i_a = 0$$

$$5 + \frac{5-(-50)}{5} + i_a = 0$$

$$i_a = -16\,A$$

Therefore, $P_{5V} = -(5)\,i_a = 80\,W$



The dependent source

Similarly, we need a KCL equation at node c to find i_c

$$\frac{V_c - V_b}{4} + \frac{V_c - V_a}{5} + i_c = 0$$

$$\frac{-50-60}{4} + \frac{-50-5}{5} + i_c = 0$$

$$i_c = 26\,A$$

Therefore, $P_{-10i_x} = (-10i_x)\,i_c = -50 \times 26$

$$= -1300\,W$$

The 10v source

Finally, a KCL equation at node b is needed

$$-i_a + i_b + \frac{V_b - V_c}{4} = 0$$

$$-(-16) + i_b + \frac{-60-(-50)}{4} = 0$$

$$i_b = -31\,A$$

Therefore, $P_{10V} = 10 \times i_b = -310\,W$

(Please do not write in this space.)

#1 *(15)*	#2 *(11)*	#3 *(14)*	Total *(40)*