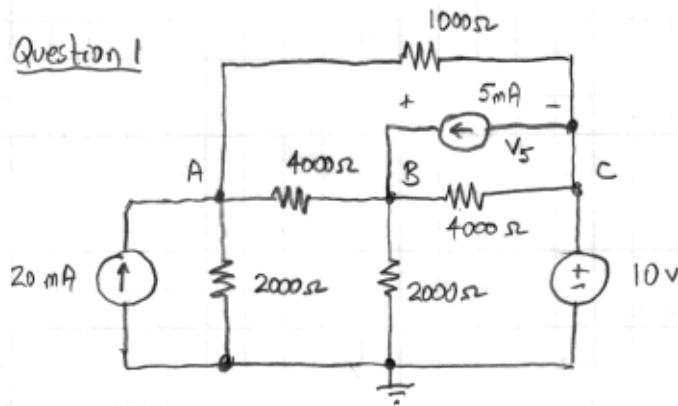


Question 1



Can use any method, but node-voltage looks easiest. Mesh-current could also be used, but it looks like there are 5 meshes (of which two are fixed by current sources). For node-voltage, there will be just two node equations for the reference node as shown.

Node C fixed at $V_C = 10\text{V}$.

$$\text{Node A: } -0.02 + \frac{V_A}{2000} + \frac{V_A - V_B}{4000} + \frac{V_A - V_C}{1000} = 0$$

$$\text{or } -80 + 2V_A + V_A - V_B + 4V_A - 4V_C = 0$$

$$7V_A - V_B - 4V_C = 80$$

$$7V_A - V_B = 120 \quad (1)$$

$$\text{Node B: } \frac{V_B - V_A}{4000} + \frac{V_B}{2000} + \frac{V_B - V_C}{4000} - 0.005 = 0$$

$$\text{or } V_B - V_A + 2V_B + V_B - V_C - 20 = 0$$

$$-V_A + 4V_B - V_C = 20$$

$$-V_A + 4V_B = 30 \quad (2)$$

Add 4 times equation (1) to (2)

$$28V_A - 4V_B = 480$$

$$\underline{-V_A + 4V_B = 30}$$

$$27V_A = 510$$

$$\text{so } V_A = 18.\bar{8} \text{ V}$$

and therefore, $V_B = 12.\bar{2} \text{ V}$. As indicated in the above diagram, V_5 is the voltage across the 5 mA current source.

$$V_5 = V_B - V_C = 12.2 - 10 = -2.2 \text{ V}$$

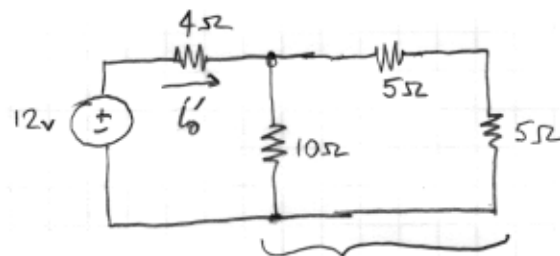
Finally, using the passive reference convention

$$P_5 = -V_5 i = -2.2 \times 0.005 = -11.1 \text{ mW (and negative, means generating power)}$$

Question 2

Let's tackle the sources one at a time from left to right.

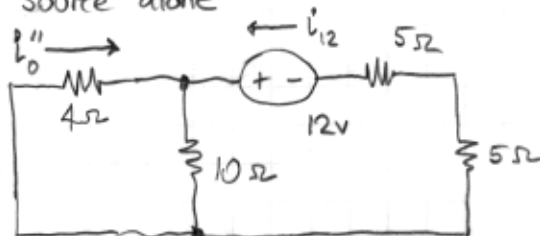
Left-most 12v source alone



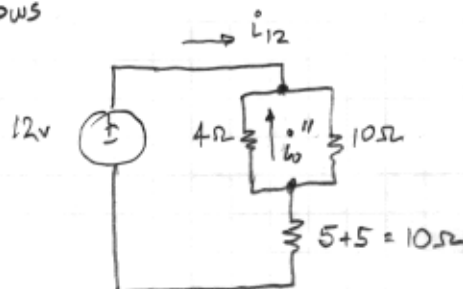
$$\text{equivalent resistance } R_{eq} = (5+5) // 10 = \frac{10 \cdot 10}{10+10} = 5\Omega$$

$$\text{Hence, } i'_0 = \frac{12}{4 + R_{eq}} = \frac{4}{3} \text{ A}$$

Middle 12v source alone



We have a current divider. We may redraw the circuit as follows



$$\text{We have } i_{12} = \frac{12}{(4 // 10) + 10}$$

$$i_{12} = \frac{12}{2.857 + 10}$$

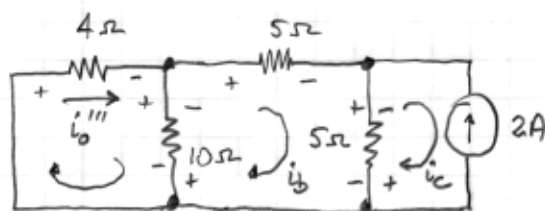
$$\text{so } i_{12} = 0.9\bar{3} \text{ A}$$

Current divider, with i_0'' pointing in the opposite direction

$$i_0'' = -\frac{10}{4+10} \times i_{12}$$

$$= -2/3 \text{ A}$$

Finally, the current source acting alone



A little more work this time. Since we need a current, let's give the mesh-current method a try.

$$\text{Mesh } i_0''': \quad 4i_0''' + 10(i_0''' - i_b) = 0$$

$$14i_0''' - 10i_b = 0 \quad (1)$$

$$\text{Mesh } i_b: \quad 10(i_b - i_0''') + 5i_b + 5(i_b - i_c) = 0$$

$$-10i_0''' + 20i_b - 5i_c = 0 \quad (2)$$

$$\text{Mesh } i_c: \quad i_c = -2A$$

$$\text{Hence, equation (2) becomes } -10i_0''' + 20i_b = -10 \quad (3)$$

Take 2 times equation (1) and add to (3)

$$28i_0''' - 20i_b = 0$$

$$-10i_0''' + 20i_b = -10$$

$$18i_0''' = -10$$

$$\text{so } i_0''' = -5/9$$

Combining all three results,

$$i_0 = i_0' + i_0'' + i_0'''$$

$$= 4/3 - 2/3 - 5/9$$

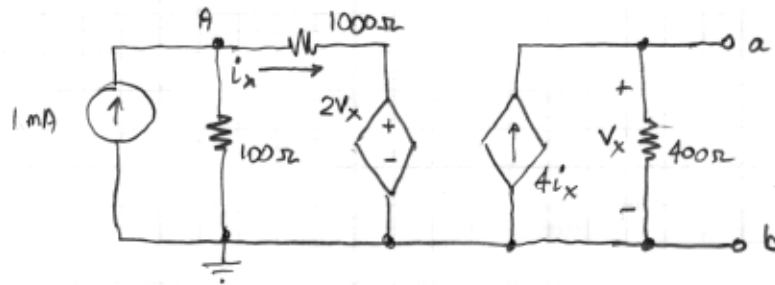
$$= 6/9 - 5/9$$

$$i_0 = 1/9 \text{ A}$$



Question 3

First, let's find the open-circuit voltage, which is V_t



First, we may observe that $V_{oc} = V_t = V_x = V_{ab}$.

In the left portion of the circuit, we can write a single node equation.

$$\text{Node A: } -0.001 + \frac{V_A}{100} + \frac{V_A - 2V_x}{1000} = 0$$

$$\text{or } -1 + 10V_A + V_A - 2V_x = 0 \quad (1)$$

$$11V_A - 2V_x = 1$$

$$\text{We also see that } i_x = \frac{V_A - 2V_x}{1000} \quad (2)$$

In the right portion of the circuit, we may immediately say

$$V_t = V_x = 400 \times 4i_x$$

Combining this with equation (2) gives

$$V_x = 400 \times 4 \left(\frac{V_A - 2V_x}{1000} \right)$$

$$= 1.6V_A - 3.2V_x$$

$$\text{so } 4.2V_x = 1.6V_A \quad (3)$$

$$\text{and } V_x = \frac{8}{21}V_A$$

Substituting (3) into (1) gives

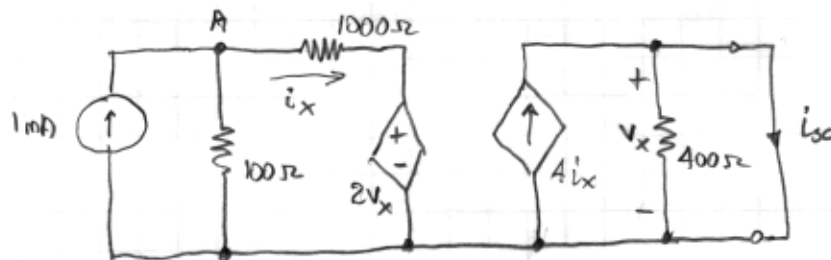
$$11V_A - 2 \left(\frac{8}{21} \right) V_A = 1$$

$$\text{hence } V_A = 0.09767 \text{ V}$$

Back to equation (3): $V_x = V_t = V_{oc} = V_{ab} = 0.09767 \left(\frac{0.1}{21} \right)$

$$V_t = 0.03721 \text{ V}$$

Now, need Thevenin resistance. Unfortunately, we have dependent sources, so no short-cuts. Find i_{sc}



This isn't so bad because we may observe $V_x = 0$ due to the short circuit. Also, all the current passes through the short circuit

$$i_{sc} = 4i_x$$

This also causes the dependent voltage source to appear as a short circuit, so that at node A

$$-0.001 + \frac{V_A}{100} + \frac{V_A}{1000} = 0$$

$$-1 + 10V_A + V_A = 0$$

$$\therefore V_A = 0.0909091 = 1/11 \text{ V}$$

Equation (2) still applies, but with $V_x = 0$;

$$i_x = \frac{V_A}{1000} = 1/11000 \text{ A}$$

And so $i_{sc} = 4i_x = 4/11000 \text{ A}$

Finally, $R_t = \frac{V_t}{i_{sc}} = \frac{0.03721}{(4/11000)}$

$$R_t = 102.335 \Omega$$

We may also compute i_x simply using a current divider.

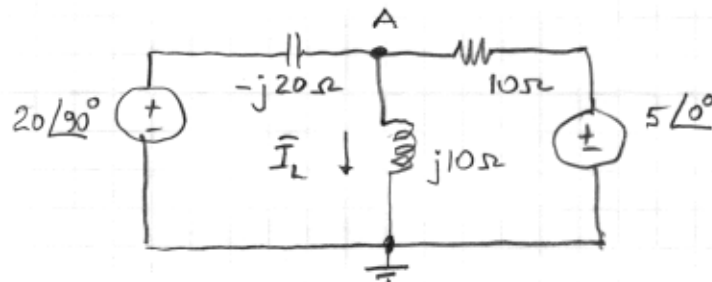
Question 4

Need impedances of the inductor and capacitor for $\omega = 10$ rad/sec

$$Z_L = j\omega L = j \times 10 \times 1 = j10\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 10 \times 0.005} = -j20\Omega$$

Hence, in terms of phasors and complex impedances, the circuit is



Choose any method. How about node-voltage:

$$\text{Node A: } \frac{\bar{V}_A - 20\angle 90^\circ}{-j20} + \frac{\bar{V}_A}{j10} + \frac{\bar{V}_A - 5\angle 0^\circ}{10} = 0$$

$$\text{or } \frac{\bar{V}_A - j20}{-j20} + \frac{\bar{V}_A}{j10} + \frac{\bar{V}_A - 5}{10} = 0$$

Multiply through by $-j20$

$$\bar{V}_A - j20 - 2\bar{V}_A - 2j(\bar{V}_A - 5) = 0$$

$$-\bar{V}_A - 2j\bar{V}_A - j20 + j10 = 0$$

$$-\bar{V}_A(1 + 2j) = j10$$

$$\text{so } \bar{V}_A = \frac{-j10}{1 + 2j}$$

And therefore, the phasor current \bar{I}_L is

$$\bar{I}_L = \frac{\bar{V}_A}{j10} = \frac{-1}{1 + 2j}$$

$$= \frac{-1}{1 + 2j} \times \frac{1 - 2j}{1 - 2j} = \frac{-1 + 2j}{5}$$

$$\begin{aligned}\bar{I}_2 &= -0.2 + j0.4 \quad (\text{second-quadrant}) \\ &= 0.4472 \angle 116.565^\circ\end{aligned}$$

Finally, $i_2(t) = 0.4472 \cos(10t + 116.565^\circ)$