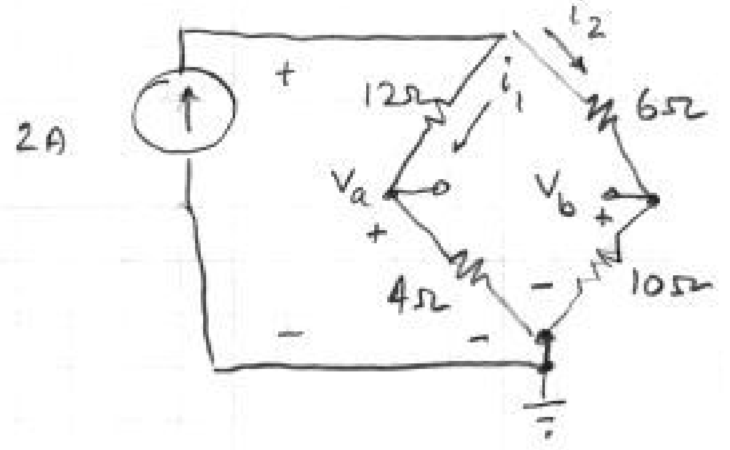
Question 1

(a) Many ways to solve this



We have two parallel branches with 16Ω in each.

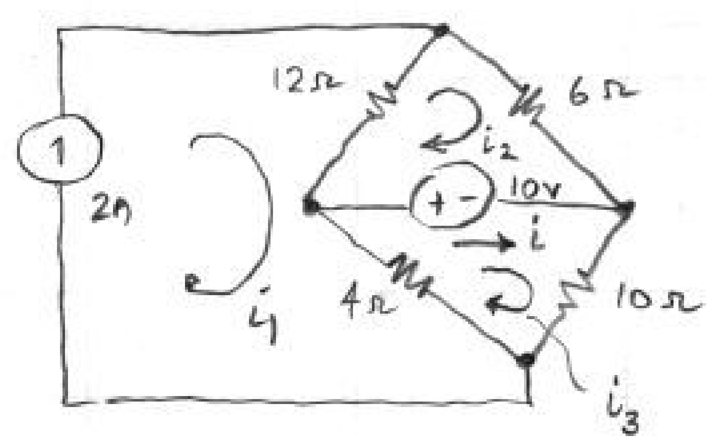
Current will split evenly

$i_1 = i_2 = 1\text{ A}$

Therefore, $V_a = 1 \times 4 = 4\text{ v}$
$V_b = 1 \times 10 = 10\text{ v}$

And $V_{ab} = V_a - V_b = -6\text{ v}$

(b) Set up for mesh-current method



We will need

$i = i_3 - i_2$

Mesh 1 : $i_1 = 2A$

Mesh 2 : $6i_2 - 10 + 12(i_2 - i_1) = 0$
$-12i_1 + 18i_2 = 10 \quad (1)$

Mesh 3 : $10i_3 + 4(i_3 - i_1) + 10 = 0$
$-4i_1 + 14i_3 = -10 \quad (2)$

Substitute for i_1

$-24 + 18i_2 = 10$
so $18i_2 = 34$ $\therefore$ $\boxed{i_2 = \frac{17}{9}} = 1.889$

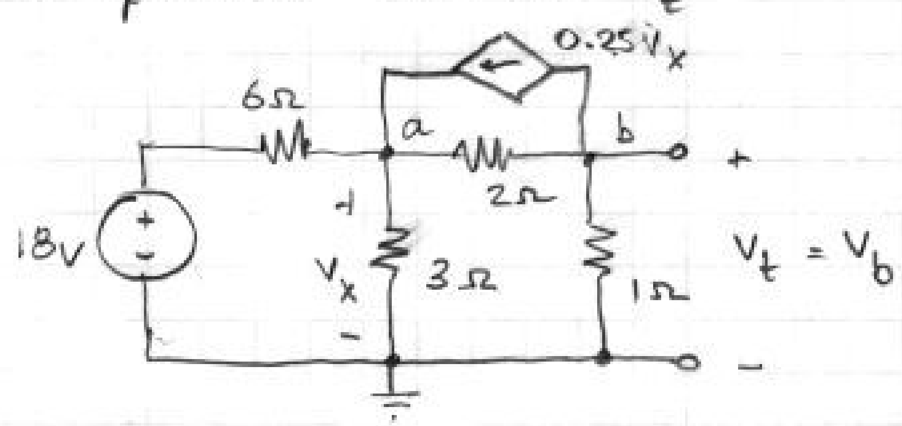
$$-8 + 14 i_3 = -10$$
$$14 i_3 = -2 \qquad \therefore \boxed{i_3 = -\frac{1}{7}} \doteq -0.1429$$

Finally, $i = i_3 - i_2 = -\frac{1}{7} - \frac{17}{9} = \frac{-9}{63} - \frac{119}{63}$

$$\boxed{i = -\frac{128}{63}} = -2.082 \text{ A}$$

Question 2

Thevenin equivalent. Solve for V_t



Node-voltage method a sensible way to proceed

Node a: $\frac{V_a - 18}{6} + \frac{V_a - V_b}{2} - 0.25 V_x + \frac{V_a}{3} = 0$

We observe that $V_x = V_a$, so

$$\frac{V_a - 18}{6} + \frac{V_a - V_b}{2} - 0.25 V_a + \frac{V_a}{3} = 0$$

$$2V_a - 36 + 6V_a - 6V_b - 3V_a + 4V_a = 0$$

$$9V_a - 6V_b = 36 \qquad (1)$$

Node b: $\frac{V_b - V_a}{2} + \frac{V_b}{1} + 0.25 V_x = 0$

$$2V_b - 2V_a + 4V_b + V_a = 0$$

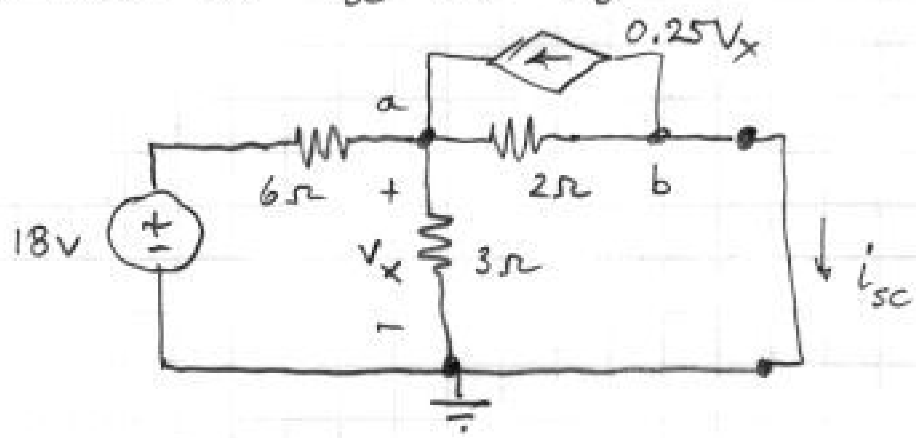
$$-V_a + 6V_b = 0 \qquad (2)$$

From (2), $V_a = 6V_b$, so from (1)

$$54V_b - 6V_b = 36$$

$$48V_b = 36 \qquad \therefore \boxed{V_t = V_b = 0.75\,\text{V}}$$

Now solve for i_{sc} and R_t



Let's stick with node-voltage in terms of unknown i_{sc}.
V_b now known as $V_b = 0$.

node a: $\dfrac{V_a - 18}{6} + \dfrac{V_a}{3} + \dfrac{V_a - V_b}{2} - 0.25V_x = 0$
(unchanged from previous calculation)

$$9V_a - \cancel{6V_b}^{\,0} = 36$$

$$V_a = 4$$

node b: $\dfrac{V_b - V_a}{2} + 0.25V_x + i_{sc} = 0$

$$\cancel{2V_b}^{\,0} - 2V_a + V_a + 4i_{sc} = 0$$

$$-V_a + 4i_{sc} = 0$$

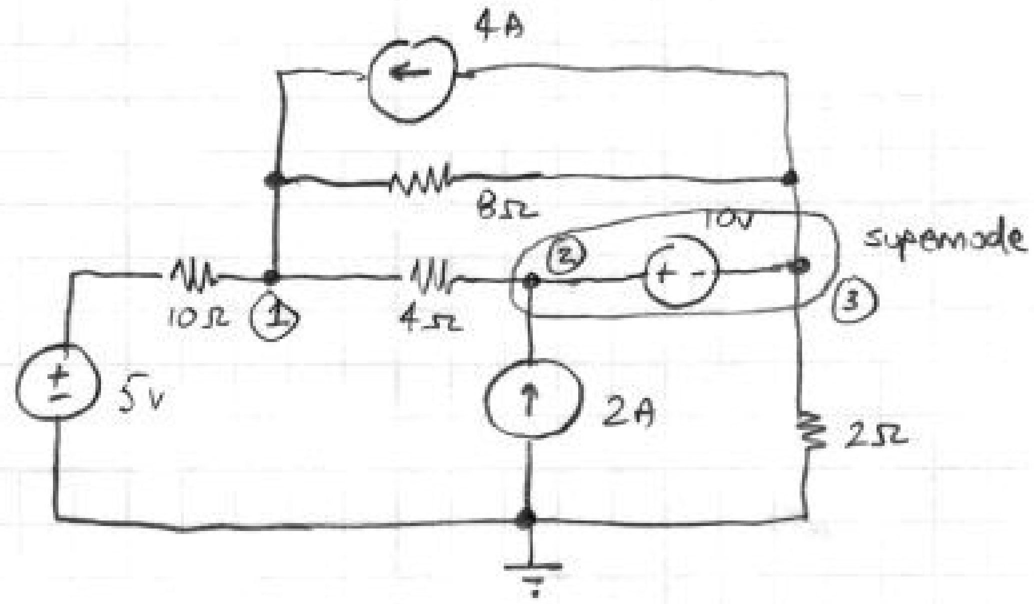
$$i_{sc} = \frac{V_a}{4} = 1$$

Therefore,

$$\boxed{\begin{array}{l} V_t = 0.75\ \text{V} \\ R_t = \dfrac{V_t}{i_{sc}} = 0.75\ \Omega \end{array}}$$

Question 3

(a) Node-voltage method



Node 1 : $\dfrac{V_1 - 5}{10} + \dfrac{V_1 - V_2}{4} + \dfrac{V_1 - V_3}{8} - 4 = 0$

$$4V_1 - 20 + 10V_1 - 10V_2 + 5V_1 - 5V_3 - 160 = 0$$

$$19V_1 - 10V_2 - 5V_3 = 180 \qquad (1)$$

Supernode : $\dfrac{V_2 - V_1}{4} - 2 + \dfrac{V_3}{2} + \dfrac{V_3 - V_1}{8} + 4 = 0$

$$2V_2 - 2V_1 - 16 + 4V_3 + V_3 - V_1 + 32 = 0$$

$$-3V_1 + 2V_2 + 5V_3 = -16 \qquad (2)$$

And within the supernode, $V_2 - V_3 = 10$
so $V_3 = V_2 - 10 \qquad (3)$

Substituting (3) into (1) and (2)

$$19V_1 - 10V_2 - 5(V_2 - 10) = 180$$
$$19V_1 - 15V_2 = 130 \qquad (4)$$

$$-3V_1 + 2V_2 + 5(V_2 - 10) = -16$$
$$-3V_1 + 7V_2 = 34 \qquad (5)$$

From (5), $V_2 = \dfrac{34 + 3V_1}{7}$, and substitute into (4)

$$19V_1 - 15\left(\frac{34+3V_1}{7}\right) = 130$$

$$133V_1 - 510 - 45V_1 = 910$$

$$88V_1 = 1420$$

so $V_1 = 16.136\text{ v}$

and therefore

$$V_2 = \frac{34 + 3(16.136)}{7}$$

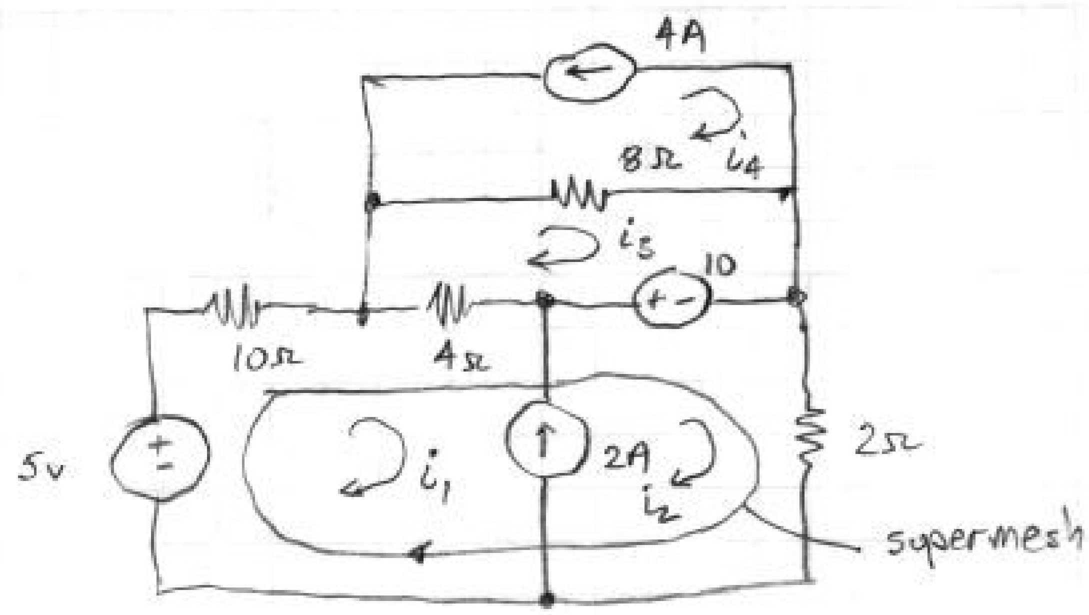
so $V_2 = 11.773\text{ v}$

We don't need it, but $V_3 = V_2 - 10 = 1.773$

Finally, $i_x = \frac{V_2 - V_1}{4}$

$$\boxed{i_x = 1.09091}$$

Alternatively, by the mesh-current method



Supermesh: $-5 + 10i_1 + 4(i_1 - i_3) + 10 + 2i_2 = 0$

$$14i_1 + 2i_2 - 4i_3 = -5 \qquad (1)$$

Mesh 3: $4(i_3 - i_1) + 8(i_3 - i_4) - 10 = 0$

$$-4i_1 + 12i_3 - 8i_4 = 10 \qquad (2)$$

Mesh 4: $i_4 = -4A$ (3)

And within the supermesh, $i_2 - i_1 = 2$

so $i_2 = 2 + i_1$ (4)

First, substitute (3) into (2)

$$-4i_1 + 12i_3 - 8(-4) = 10$$
$$-4i_1 + 12i_3 = -22 \qquad (5)$$

Substitute (4) into (1)

$$14i_1 + 2(2+i_1) - 4i_3 = -5$$
$$16i_1 - 4i_3 = -9 \qquad (6)$$

Add 4×(5) with (6)

$$-16i_1 + 48i_3 = -88$$
$$16i_1 - 4i_3 = -9$$
$$44i_3 = -97$$

$$\therefore i_3 = -2.2045$$

From (6),

$$i_1 = \frac{4i_3 - 9}{16} = \frac{4(-2.2045) - 9}{16}$$

$$\therefore i_1 = -1.1136$$

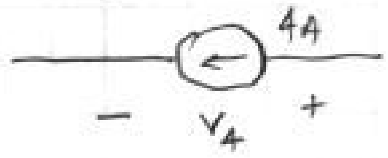
Don't need 'em but

$$i_2 = 0.8864$$
$$i_4 = -4$$

And finally $i_x = i_1 - i_3$

$$\boxed{i_x = 1.09091}$$

(b) Using node voltages V_1, V_3, and assigning the polarity as follows

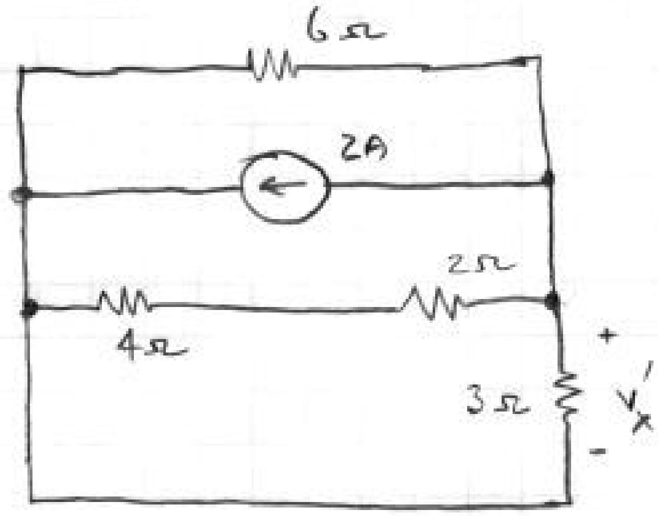


We have $V_4 = V_3 - V_1 = -14.3634\ \text{v}$

and $\boxed{P = vi = -57.4\ \text{W}}$ GENERATING

Question 4

Solve by superposition. Let's start with the 2A source



It appears that the total resistance across the source is

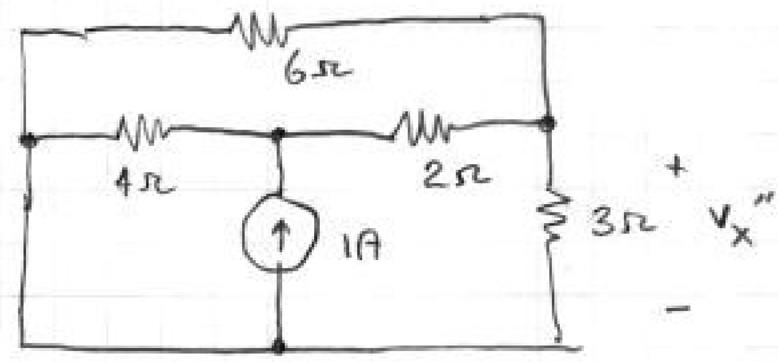
$$R_{eq} = 6 \parallel (2+4) \parallel 3$$

and that V_x' is the total voltage across the source.

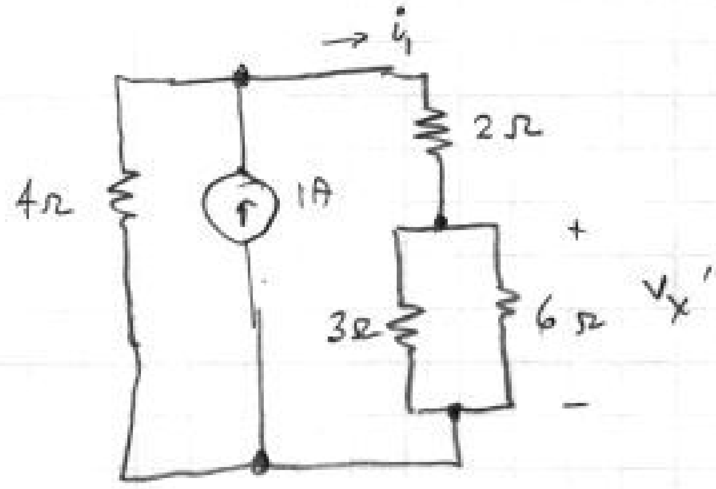
$$\begin{aligned} R_{eq} &= 6 \parallel 6 \parallel 3 \\ &= 3 \parallel 3 \\ &= 1.5\Omega \end{aligned}$$

and therefor, $$\begin{aligned} V_x' &= -2A \times 1.5\Omega \\ &= -3\ \text{V} \end{aligned}$$

Next, the 1A source acting alone



Redrawing,

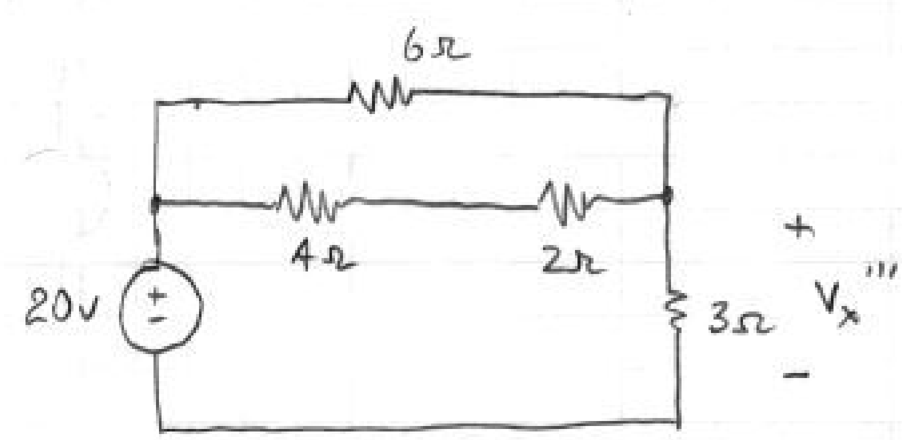


On the right side, $R_{eq} = 2 + 6 /\!/ 3$
$= 4\Omega$

Current divider, so $i_1 = \dfrac{4}{4+4} \times 1 = 0.5\text{ A}$

Therefore, $V_x'' = 0.5 \times 3 /\!/ 6$
$= 1\text{ V.}$

Finally, the 20v source acting alone



Voltage divider

$$V_x''' = \frac{3}{3 + (6//(2+4) + 3)} \times 20$$

$$= 10\,V$$

Combining all three results,

$$V_x = V_x' + V_x'' + V_x'''$$

$$= -3 + 1 + 10$$

$$\boxed{V_x = 8\,V}$$