PIMS Summer School on *Inclusive Paths to Number Theory*
August 23-27, 2021, hosted by University of Calgary
An Introduction to Function Fields

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Website: `https://people.ucalgary.ca/rscheidl/PIMS-SummerSchool.html`

Overview

This is a first introduction to algebraic function fields. Topics include discrete valuations, function fields, divisors and genus, function fields of genus 0, 1 and 2. Time permitting, other topics will be covered.

Assumed background knowledge

- Polynomials, especially over finite fields.
- Basic abstract algebra: groups, rings, fields, ideals, prime ideals.
- Knowledge of algebraic number theory as covered in a typical course devoted to that subject is an asset, especially familiarity with Dedekind domains, number field extensions, rings of integers and decomposition of ideals into prime ideals.

Sources and material covered

I will be lecturing from slides. A preliminary set of slides is posted on the website.

The material is primarily taken from the sources below. If your institutional library subscribes to SpringerLink, you should be able to obtain PDF versions of all these books for free; even if it doesn't, you may be able to get copies off the internet.

1. Henning Stichtenoth, *Algebraic Function Fields and Codes*, second edition, Springer 2009
   - Sections 1.1, 1.2, 1.4 (up to and including Def. 1.4.3)
   - Sections 3.1, 3.3 (Theorem 3.3.7 only), 3.4 (Theorem 3.4.13 only), 3.5 (Theorem 3.5.1 only), 3.6
   - Sections 6.1, 6.2

2. Joseph Silverman, The Arithmetic of Elliptic Curves, second edition, Springer 2009

   – Sections III.2 (up to and excluding singular Weierstrass Equations) and III.3

3. Alfred J. Menezes, Yi-Hong Wu and Robert J. Zuccherato, An elementary introduction to hyperelliptic curves, CORR 96-19, University of Waterloo 1996 (also appeared as an appendix in: Neal Koblitz, *Algebraic Aspects of Cryptography*, Algorithms and Computation in Mathematics, vol. 3. Springer, Berlin, 1998).

4. Steven D. Galbraith, Michael Harrison and David J. Mireles Morales, Efficient hyperelliptic arithmetic using balanced representation for divisors. In *Algorithmic Number Theory*, Lecture Notes in Comput. Sci., vol. 5011, Springer, Berlin, 2008, 342–356.

Good supplementary sources on function fields are

5. Gabriel D. Villa Salvador, Topics in the Theory of Algebraic Function Fields, Birkhäuser 2006 (we cover Sections 1.1-1.2, 2.1-2.4, 3.1-3.2 and Chapter 4)

6. Michael Rosen, Number Theory in Function Fields, Springer 2002

Exercises

A set of exercises of varying difficulty accompanying the lectures is posted on the website. Many of these exercises are facts stated (but not proved) during the lectures, along with other supporting results that you are asked to prove here.