Factual content in an algorithmic theory of meaning

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BIRS, February 2007

Theory of Referential Intensions

Meaning as the algorithm that computes the denotation

Sentence A	$\mapsto$	(Global) Meaning $\mathrm{int}(A)$	$\mapsto$	Denotation $\mathrm{den}(A)$
Utterance at a $A(\bar{a})$	$\mapsto$	Local Meaning at a $\mathrm{int}(A(\bar{a}))$	$\mapsto$	Denotation at a $\mathrm{den}(A(\bar{a}))$

***Factual Content at** a*

Information that a Sentence Conveys about the World

Syntax of $L_{ar}^{\lambda}(K)$

- Types: $\tau :\equiv \mathsf{e} \mid \mathsf{t} \mid \mathsf{s} \mid (\tau_1 \to \tau_2)$
- K : finite set of constants with types

$$\tilde{\sigma} :\equiv (\mathsf{s} \to \mathsf{e}) \mid (\mathsf{s} \to \mathsf{t}) \mid \tilde{\sigma}_1 \to \tilde{\sigma}_2$$

EXAMPLES

John : $\tilde{\mathsf{e}}$, he : $\tilde{\mathsf{e}}$, love : $\tilde{\mathsf{e}} \times \tilde{\mathsf{e}} \to \tilde{\mathsf{t}}$, Necessarily : $\tilde{\mathsf{t}} \to \tilde{\mathsf{t}}$

- Terms: $A :\equiv \mathsf{c} \mid x \mid B(C) \mid \lambda(v)B \mid$
  A_0 where $\{p_1 := A_1, \ldots, p_n := A_n\}$

EXAMPLES

John loves Mary $\xrightarrow{\text{render}}$ $\mathsf{love}(\mathsf{John}, \mathsf{Mary})$
John loves himself $\xrightarrow{\text{render}}$ $\lambda(x)\big(\mathsf{love}(x, x)\big)(\mathsf{John})$,
$\mathsf{love}(p, p)$ where $\{p := \mathsf{John}\}$

(Global) Meaning

- Denotational Semantics: $A : \tau \mapsto \textbf{den}(A)(g) \in \mathbb{T}_\tau$

- Reduction Relation ($A \Rightarrow B$) defined by ten reduction rules
- For each term A, there is a unique (up to congruence) irreducible recursive term $\mathrm{cf}(A) \equiv A_0$ where $\{p_1 := A_1, \ldots, p_n := A_n\}$ such that $A \Rightarrow \mathrm{cf}(A)$.

EXAMPLES

$\mathrm{love}(\mathrm{John}, \mathrm{Mary}) \Rightarrow_{\mathrm{cf}} \mathrm{love}(p, q)$ where $\{p := \mathrm{John}, q := \mathrm{Mary}\}$

$\lambda(x)(\mathrm{love}(x, x))(\mathrm{John}) \Rightarrow_{\mathrm{cf}} \lambda(x)(\mathrm{love}(x, x))(p)$ where $\{p := \mathrm{John}\}$

- **Referential Intension of** A $(\textbf{int}(A))$: tuple of functions defined by $\mathrm{cf}(A)$.

Referential Synonymy

The canonical form of A defines formally within the language the meaning of A

A is **referentially synonymous** with B $\quad (A \approx B)$ if and only if

$$A \Rightarrow_{\text{cf}} A_0 \text{ where } \{p_1 := A_1, \ldots, p_n := A_n\}$$
$$B \Rightarrow_{\text{cf}} B_0 \text{ where } \{p_1 := B_1, \ldots, p_n := B_n\}$$
$$\text{and} \quad \forall g, \text{den}(A_i)(g) = \text{den}(B_i)(g) \quad (i = 0, \ldots, n)$$

EXAMPLES

$\text{love}(p, p) \text{ where } \{p := \text{John}\} \approx \lambda(x)(\text{love}(x, x))(p) \text{ where } \{p := \text{John}\}$

$\text{love}(p, p) \text{ where } \{p := \text{John}\} \not\approx \text{love}(p, p) \text{ where } \{p := \text{he}\}$

Local Meaning

- **Local Meaning of** $A : \tilde{\mathrm{t}}$ **at state** a: $\mathrm{cf}(A(\bar{a}))$
- A at state a is **locally synonymous** with B at state b if and only if $A(\bar{a}) \approx B(\bar{b})$.

EXAMPLE 1 - $\forall p, love(p,p)(a) = hate(p,p)(a)$

$$\mathrm{love}(p,p)(\bar{a}) \text{ where } \{p := \mathrm{John}\} \approx \mathrm{hate}(p,p)(\bar{a}) \text{ where } \{p := \mathrm{John}\}$$

EXAMPLE 2 - $John(a) = he(a)$

$$\mathrm{love}(p,p)(\bar{a}) \text{ where } \{p := \mathrm{John}\} \not\approx \mathrm{love}(p,p)(\bar{a}) \text{ where } \{p := \mathrm{he}\}$$

What is Said about the World

Local associate f_* of an object f

- Locality: $love : \tilde{\mathrm{e}} \times \tilde{\mathrm{e}} \to \tilde{\mathrm{t}}$
- "Unfolding" of types: $\tilde{\sigma} \equiv \tilde{\sigma}_1 \times \ldots \times \tilde{\sigma}_n \to \tilde{\sigma}_0$ where $\tilde{\sigma}_0 \equiv \tilde{\mathrm{e}}$ or $\tilde{\mathrm{t}}$.

Definition

$f : \tilde{\mathrm{e}}$ or $\tilde{\mathrm{t}}$, then $\quad f_* = f$.
$f : \tilde{\sigma}_1 \times \ldots \times \tilde{\sigma}_n \to \tilde{\sigma}_0$, then $\quad f_* : \mathrm{s} \times \sigma_1 \times \ldots \times \sigma_n \to \sigma_0$ such that

$$f(f_1, \ldots, f_n, a) = f_*\big(a, (f_1)_*(a), \ldots, (f_n)_*(a)\big).$$

EXAMPLE

$love_* : \mathrm{s} \times \mathrm{e} \times \mathrm{e} \to \mathrm{t}$ and for any $f_1, f_2 : \tilde{\mathrm{e}}$, $a : \mathrm{s}$,

$$love\big(f_1, f_2, a\big) = love_*\big(a, (f_1)_*(a), (f_2)_*(a)\big) = love_*(a, f_1(a), f_2(a)).$$

A, state variable $u \mapsto A^{*,u}$

- If for all c : $\tilde{\sigma} \in K$ are such that $c \in \mathbb{T}_{\tilde{\sigma}}$ is local, then for all closed terms A, $\mathrm{den}(A)$ is local.

Question

Is it possible to define formally for local terms $\big(\mathrm{den}(A)\big)_*$?

- $A : \tilde{\sigma}$, state variable $u \quad \mapsto \quad A^{*,u} : \sigma$
  - For closed A, $A^{*,u}$ defines the associate of A at u, and
  - the $*$-transformation respects the reduction calculus
- in an extension of $\mathrm{L}^{\lambda}_{\mathrm{ar}}(K)$ with **Associate Application**

EXAMPLE

$\big(\mathrm{love}(p,p) \text{ where } \{p := \mathrm{John}\}\big)^{*,u} \equiv \mathrm{love}[u](q,q) \text{ where } \{q := \mathrm{John}[u]\}$

Factual Content

- A at state $a \quad \mapsto \quad A^{*,\bar{a}} \quad \mapsto \quad \mathrm{cf}(A^{*,\bar{a}})$
- **Factual content of** A **at state** a: $\mathrm{cf}(A^{*,\bar{a}})$
- A at state a is **factually synonymous** with B at state b if and only if $A^{*,\bar{a}} \approx B^{*,\bar{b}}$.

EXAMPLE- $John(a) = he(a)$

$\mathrm{love}[\bar{a}](q, q)$ where $\{q := \mathrm{John}[\bar{a}]\} \approx \mathrm{love}[\bar{a}](q, q)$ where $\{q := \mathrm{he}[\bar{a}]\}$

Theorem

$A \approx B \iff \lambda(u)A^{*,u} \approx \lambda(u)B^{*,u}$

Factual Content for arbitrary terms

- Necessarily : $\tilde{t} \to \tilde{t}$, former : $(\tilde{e} \to \tilde{t}) \times \tilde{e} \to \tilde{t}$
- Tracing locality – **locality input index** ℓ
- Associate $(f)^{\ell}_{*}$ of an object f with respect to a closed locality input index ℓ.

EXAMPLE

$(\textit{Necessarily})^{\ell}_{*} : s \times \tilde{t} \to t$ with $\ell \equiv \langle I \hookrightarrow 1 \rangle$
$\textit{Necessarily}(f_1, a) = (\textit{Necessarily})^{\ell}_{*}(a, (f_1)^{I}_{*}) = (\textit{Necessarily})^{\ell}_{*}(a, f_1)$

- Formal definition of $\big(\mathrm{den}(A)\big)^{\ell}_{*}$ in L^{λ}_{ar} using the formation tree of A suitably labeled with locality indices (**locality proof**)
- Work in Progress: Uniqueness, Compositionality

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$A^{*,\bar{a}}$	$\mapsto$	Factual Content at a $\text{int}(A^{*,\bar{a}})$	$\mapsto$	$\text{den}(A^{*,\bar{a}})$