



# Banff International Research Station

for Mathematical Innovation and Discovery

## Mathematical Methods in Philosophy

February 18–23, 2007

### MEALS

\*Breakfast (Buffet): 7:00–9:00 am, Donald Cameron Hall, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

### SCHEDULE

#### Sunday, February 18

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
**17:30–19:30** Buffet Dinner, Donald Cameron Hall  
**20:00** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and small assortment of snacks available on a cash honour-system.

#### Monday, February 19

- 7:00–8:45** Breakfast
- 8:45–9:00** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
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- 9:00–10:15** **Chair:** Richard Zach  
**Branden Fitelson** (University of California, Berkeley)  
*Survey on formal epistemology*
- 10:15–10:35** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:35–11:30** **Delia Graff Fara** (Princeton University)  
*Relative identity and de re modality*
- 11:30–14:00** Lunch
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- 14:00–14:55** **Chair:** Aldo Antonelli  
**JC Beall** (University of Connecticut) and  
**Michael Glanzberg** (University of California, Davis)  
*Truth and paradox*
- 15:00–15:30** **Kai Wehmeier** (University of California, Irvine)  
*Identity is not a relation*
- 15:30–16:00** Coffee Break, 2nd floor lounge, Corbett Hall
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- 16:00–16:55** **Chair:** Bill Tait  
**Yiannis Moschovakis** (UCLA)  
*Synonymy*
- 17:00–17:55** **Byeong-Uk Yi** (University of Toronto)  
*Is logic axiomatizable?*

18:00–19:30 Dinner

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**Chair:** JC Beall  
19:30–20:30 Graduate Student Presentations  
**Kenny Easwaran** (University of California, Berkeley)  
**Eleni Kalyvianaki** (University of Athens)  
**Kate Manne** (MIT)  
**Rafał Urbaniak** (University of Calgary)

## Tuesday, February 20

7:00–9:00 Breakfast

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**Chair:** Alasdair Urquhart  
9:00–10:15 **Marcus Kracht** (UCLA)  
*The certain past and possible future of modal logic*  
10:15–10:35 Coffee Break, 2nd floor lounge, Corbett Hall  
10:35–11:30 **Steve Awodey** (Carnegie Mellon University)  
*Topological semantics for first-order modal logic* (joint work with Kohei Kishida)  
11:30–13:00 Lunch  
13:00–13:30 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

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**Chair:** Solomon Feferman  
13:30–14:25 **Volker Halbach** (University of Oxford)  
*The Kripke-Feferman theory of truth*  
14:30–15:25 **Jeff Ketland** (University of Edinburgh)  
*Truth and reflection*  
15:25–16:00 Coffee Break, 2nd floor lounge, Corbett Hall  
**Chair:** Philip Ebert  
16:00–16:55 **Hannes Leitgeb** (University of Bristol)  
*Applications of mathematics in philosophy: four case studies*  
17:00–17:55 **Gabriel Uzquiano** (University of Oxford)  
*Ineffability and reflection* (joint work with Stewart Shapiro)

18:00–19:30 Dinner

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**Chair:** Graham Priest  
19:30–20:25 **Solomon Feferman** (Stanford University)  
*A nice formal theory non-hierarchical theory of truth*

## Wednesday, February 21

7:00–9:00 Breakfast

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**Chair:** Agustín Rayo  
9:00–10:15 **Harvey Friedman** (Ohio State University)  
*Concept calculus*  
10:15–10:35 Coffee Break, 2nd floor lounge, Corbett Hall  
10:35–11:30 **Tim Williamson** (University of Oxford)  
*Adding probabilities to epistemic logic*  
11:30 Group Photo; meet on the front steps of Corbett Hall  
11:30–13:30 Lunch  
Free Afternoon

17:30–19:30 Dinner

## Thursday, February 22

7:00–9:00 Breakfast

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**Chair:** Jeff Ketland

9:00–10:15 **Stewart Shapiro** (Ohio State University)

*Life on the ship of Neurath: mathematics in the philosophy of mathematics*

10:15–10:35 Coffee Break, 2nd floor lounge, Corbett Hall

10:35–11:30 **Grigori Mints** (Stanford University)

*Effective content of non-effective proofs*

11:30–13:30 Lunch

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**Chair:** Greg Restall

13:30–14:25 **Wilfried Sieg** (Carnegie Mellon University)

*Church without dogma: axioms for computability*

14:30–15:25 **Gillian Russell** (Washington University, St. Louis)

*One true logic?*

15:25–16:00 Coffee Break, 2nd floor lounge, Corbett Hall

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**Chair:** Gillian Russell

16:00–16:55 **Greg Restall** (University of Melbourne)

*Modal models for Bradwardine's theory of truth*

17:00–17:55 **Graham Priest** (University of Melbourne)

*Many-valued modal logic*

18:00–19:30 Dinner

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**Chair:** Volker Halbach

19:30–20:25 **Eric Pacuit** (University of Amsterdam)

*Quantified Classical Modal Logic and Applications*

(joint work with Horacio Arlo-Costa)

## Friday, February 23

7:00–9:00 Breakfast

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**Chair:** Paddy Blanchette

9:00–9:55 **Dave DeVidi** (University of Waterloo)

*Non-constructive uses of constructive logics*

9:55–10:20 Coffee Break, 2nd floor lounge, Corbett Hall

10:20–11:25 **Ray Jennings** (Simon Fraser University)

*An axiomatization of family resemblance*

11:30–13:30 Lunch

**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



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## Mathematical Methods in Philosophy

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### ABSTRACTS

Speaker: **Steve Awodey** (Carnegie Mellon University)

Title: *Topological semantics for first-order modal logic*

Abstract: A new theorem is presented, which extends Tarski's classical topological completeness result from propositional to first-order S4 modal logic. (Joint work with Kohei Kishida.)

Speakers: **JC Beall** (University of Connecticut) and **Michael Glanzberg** (University of California, Davis)

Title: *Truth and paradox*

Abstract: This talk aims to be a big-picture sketch of truth and paradox – chiefly, the Liar (but also related truth-theoretic paradoxes). Our aim is not to give the details of approaches or recent theories, but rather only sketch various approaches within a broad, philosophical picture – the familiar ‘struggle’ between inconsistency and incompleteness. The desideratum of ‘completeness’ is to express one’s own semantic predicates within one’s own language. The desideratum of ‘consistency’ is to do as much consistently (or, at least, coherently, in some sense).

Approaches to the Liar that we will mention are all marked by the ways they navigate between completeness and consistency. Some key examples of these approaches include those which:

- Reconsider logic:
  1. Paraconsistent: the Liar teaches us that EFQ fails, that some sentences are true and false, but our language is nonetheless non-trivial (i.e., some sentences are ‘just true’).
  2. Paracomplete: the Liar teaches us that LEM fails, in some way that avoids a variant Liar which reinstates the paradox.
- Reconsider the semantics:
  1. Contextual: the Liar teaches us that truth is contextually sensitive, shifting the extension of ‘true’ from context to context.
  2. Revision Theory: the Liar teaches us that ‘true’ is governed by *rules of revision*.

Each of these options seeks to reject some portion of consistency or completeness, and yet present a coherent and appealing environment in which logic and semantics can coherently proceed.

Speaker: **Dave DeVidi** (University of Waterloo)

Title: *Non-constructive uses of constructive logics*

Abstract: While intuitionistic and other constructive logics have their first home in foundations of mathematics, their appearance in, for instance, metaphysical debates apart from mathematics is familiar, thanks to the work of philosophers such as Michael Dummett. In such cases, the reasons offered for supposing that constructive logic is correct are recognizably akin to those offered by mathematical constructivists. In

recent times, though, it has become increasingly common to see versions of constructive logic advocated for philosophical purpose—as part of a solution to a paradox, for instance—when no appeal to constructivist motivations is offered and no plausible one seems possible. It is not uncommon to see such proposals rejected on the grounds of incompatibility with constructivism (“no intuitionist could consistently say *that*”). This objection is beside the point if the appeal to constructive logic has some suitable non-constructivist motivation—for then the name “intuitionistic logic” (e.g.) becomes a historical curiosity, instead of an indication of who may appeal to that logic as the correct one. This response is often claimed, but seldom defended. In this talk I will briefly describe some cases of this sort, and consider the prospects for giving a non-constructivists but philosophically satisfactory defense of the claim that some-or-other constructive logic is the correct one for certain purposes.

Speaker: **Delia Graff Fara** (Princeton University)

Title: *Relative identity and de re modality*

Abstract: My goal is to defend the materialist thesis that material things are identical to the matter that composes them, by appealing to the semantic view that names are predicates; and by proposing and investigating a version of David Lewis’s counterpart theory that appeals, rather than to Lewis’s own modified similarity relation, to relations of *relative* identity in the analysis of *de re* temporal and modal claims. This is carried out in the context of a metaphysics that’s both actualist and three-dimensionalist.

Speaker: **Branden Fitelson** (University of California, Berkeley)

Title: *Survey on formal epistemology*

Abstract: What is Formal Epistemology (FE)? How does FE relate to existing areas of philosophy and logic? What are some examples of recent work in FE? In this talk, I will try to suggest some (tentative) answers to these questions. I will also discuss three concrete examples of the recent impact of FE: (1) the annual formal epistemology workshops (FEW), (2) a forthcoming special issue of *Studia Logica* that will be devoted to FE (which is part of a more general change in scope for that journal), and (3) a forthcoming change in scope for the *Stanford Encyclopedia of Philosophy* (involving FE). My hope is to give people a sense of what the fuss is all about, and also to encourage students and researchers to contribute to this burgeoning area.

Speaker: **Harvey Friedman** (Ohio State University)

Title: *Concept calculus*

Abstract: Concept Calculus provides an unexpected exact correspondence between ordinary everyday thinking about ordinary everyday things and abstract mathematics.

As an example, we identify a large range of principles involving just the two informal binary relations “better than” and “much better than,” which give rise to a variety of formal systems which are mutually interpretable with a variety of standard formal systems from logic whose strengths range from weak arithmetics to various large cardinals.

It appears that an enormous range of informal concepts lend themselves to closely related investigations. For example, we have developed a kind of naive physics based on informal notions of time and space and point mass, which also corresponds, by mutual interpretation, to these same formal systems from logic.

The hope is that concept calculus can serve as a tool for organizing and analyzing metaphysical concepts that is in rough analogy with the way that the Newton/Leibniz calculus serves as a tool for organizing and analyzing physical concepts.

Speaker: **Volker Halbach** (University of Oxford)

Title: *The Kripke-Feferman theory of truth*

Abstract: Feferman proposed to axiomatize Kripke’s theory of truth in classical logic. The resulting theory is called the Kripke-Feferman (KF) theory of truth. I argue that this theory introduces some unwanted features because it relies on classical logic, and that Kripke’s theory should be axiomatised in partial logic.

It has been argued by Reinhardt that nevertheless KF may be taken as a tool for generating theorems of a theory of truth in partial logic by focusing on those sentences  $A$  that can be proved to be true in KF. I shall argue that this justification of KF fails, as a natural axiomatisation of Kripke's theory in partial logic is proof-theoretically much weaker than the theory generated by KF. I shall also consider how these results impinge on Feferman's use of KF as the reflective closure of PA.

Speaker: **Ray Jennings** (Simon Fraser University)

Title: *Title t.b.a.*

Abstract: Abstract Text

Speaker: **Jeff Ketland** (University of Edinburgh)

Title: *Truth and reflection*

Abstract: Say that a truth theory is deflationary if, when added to any of a suitable class of base theories, the result is a conservative extension. Say that a truth theory is reflective if, when added to any of a suitable class of base theories, the reflection principles for that theory become theorems. Reflective truth theories are desirable; for, just as when we accept a statement  $A$ , we should accept "A is true," similarly, when we accept a theory  $T$ , we should accept "All axioms of  $T$  are true." Stewart Shapiro (1998) and I (1999) noted that these conditions are incompatible: reflective truth theories are non-conservative, and thus non-deflationary. In particular, Tarski's compositional theory of truth is reflective (as are more sophisticated "self-applicative" truth theories, such as the Kripke-Feferman theory), and thus non-deflationary. It seems correct to conclude then that deflationism about truth is incompatible with results in mathematical logic. Several authors (Field, Azzouni, Halbach and Tennant) have presented responses to this argument against deflationism. In this talk, I will survey these responses and offer some replies.

Speaker: **Marcus Kracht** (UCLA)

Title: *The certain past and possible future of modal logic*

Abstract: The origins of modal logic are somewhere in philosophy. However, for more than fifty years there is also a more 'technocratic' approach to the field that applies mathematical methods. Over time, it has created its own terminology and, inevitably, its own problems that it likes to deal with. Other areas of application have also been found, for example computer science. While the techniques and results for propositional modal logic are by now fairly widely known even outside the circle of mathematicians, in the domain of modal predicate logic there still is some lack of knowledge transfer between philosophers and mathematicians. I shall outline the past developments of modal logic with special attention to modal predicate logic, where I think the greatest promise for 'technocratic' modal logic is still to be found. If I am right, the new developments in modal predicate logic may help to understand the central problems of modal discourse in a new way.

Speaker: **Hannes Leitgeb** (University of Bristol)

Title: *Applications of mathematics in philosophy: four case studies*

Abstract: As we all know, mathematical methods are of crucial importance in science. Many of us believe that mathematics will play a similar role for philosophy once philosophical theories have reached a sufficient degree of complexity; to some degree, this has already happened. I try to support this thesis by stating four examples which are chosen (conveniently) from my own work:

1. Similarity, Properties, and Hypergraphs
2. Nonmonotonic Logic and Dynamical Systems
3. Belief Revision for Conditionals and Arrow's Theorem
4. Semantic Paradoxes and Non-sigma-Additive Probability Measures

Speaker: **Grigori Mints** (Stanford University)

Title: *Effective content of non-effective proofs*

Abstract: Methods of proof theory allow to extract effective bounds from some non-effective proofs and point out possibilities of obtaining sharper bounds from mathematical proofs, sometimes depending of fewer parameters. We survey several such applications and illustrate the approach for a proof of Herbrand's theorem using compactness. A new cut elimination method (in particular a new proof of Herbrand's Theorem) is obtained here by "proof mining" (unwinding) from the familiar non-effective proof. That proof begins with extracting an infinite branch when the canonical search tree for a given formula of first order logic is not closed. Our reduction of a cut does not introduce new cuts of smaller complexity preserving instead only one of the branches.

Speaker: **Yiannis Moschovakis** (UCLA)

Title: *Synonymy*

Abstract: According to a 1906 letter to Husserl, Frege believed at that time that "we must have an objective criterion for recognizing a thought as the same thought, since without such a criterion a logical analysis is not possible". So *is synonymy (sense identity) decidable?* Or, equivalently (as Frege would likely concede), *is the relation of faithful translation between sentences in two different languages decidable?*

My aims in this talk are to discuss (briefly) some historical approaches to synonymy, and (primarily) to focus on the mathematical problems of decidability which arise when senses are modelled rigorously in formalized fragments of language. About half of the talk will be dedicated to an exposition of the theory of referential intensions, by which (in slogan form) *the sense of a term is the natural algorithm which determines its denotation*. This modelling of meanings leads to both theorems and difficult open problems in the logic of synonymy.

Speaker: **Eric Pacuit** (University of Amsterdam)

Title: *Quantified Classical Modal Logic and Applications*

Abstract: I will begin by introducing and motivating the study of classical systems of first-order modal logic. In particular, I will focus on the study of neighborhood frames with constant domains and offer a series of new completeness results for salient classical systems of first order modal logic. I will then discuss general first-order neighborhood and offer a general completeness result for all classical systems of first-order modal logic. Finally, I will discuss how to extend this analysis to freely quantified classical modal logic. (Joint work with Horacio Arlo-Costa.)

Speaker: **Graham Priest** (University of Melbourne)

Title: *Many-valued modal logic*

Abstract: In standard modal logics, the worlds are two valued. There is no reason why this has to be the case, however: the worlds could be many-valued. This talk looks at many-valued modal logics. We start with the general structure of a many-valued modal logics. To illustrate this, I look briefly at modal logic based on Lukasiewicz continuum-valued logic. I then turn to one particular many-valued modal logic in more detail, modal First Degree Entailment (*FDE*). Tableaux for it and its special cases ( $K_3$  and *LP*) are provided. Modal many-valued logics engage with a number of philosophical issues. The final part of the talk illustrates by looking at one such: the issue of future contingents.

Speaker: **Greg Restall** (University of Melbourne)

Title: *Modal models for Bradwardine's theory of truth*

Abstract: I introduce Stephen Read's reconstruction of Bradwardine's theory of truth, and provide it with a simple model theory. This model theory can be used to provide a fixed-point construction to extend any classical theory with a Bradwardine truth predicate which diverges from Tarskian truth only on ungrounded sentences.

Speaker: **Gillian Russell** (Washington University, St. Louis)

Title: *One true logic?*

Abstract: In their 2006 book *Logical Pluralism*, Beall and Restall argue that there is more than one correct logic. In this paper I examine that claim and present a different argument for a similar view.

Speaker: **Stewart Shapiro** (Ohio State University)

Title: *Life on the ship of Neurath: mathematics in the philosophy of mathematics*

Abstract: This talk provides an idiosyncratic survey of the use of mathematics to support or otherwise assess programs in the philosophy of mathematics. It covers the “big three” views that dominated thinking in the early decades of the twentieth century: formalism, intuitionism, and logicism, and then moves onto contemporary descendants of these views: *ante rem* structuralism, Scottish neo-logicism, fictionalism, and various reconstructive nominalisms.

Speaker: **Wilfried Sieg** (Carnegie Mellon University)

Title: *Church without dogma: axioms for computability*

Abstract: Church’s and Turing’s theses dogmatically assert that an informal notion of computability is captured by a particular mathematical concept. I present an analysis of computability that leads to precise concepts, but dispenses with theses.

To investigate computability is to analyze processes that can in principle be carried out by calculators. Drawing on this lesson we owe to Turing and recasting work of Gandy, I formulate finiteness and locality conditions for two types of calculators, human computing agents and mechanical computing devices; the distinctive feature of the latter is that they can operate in parallel.

The analysis leads to axioms for discrete dynamical systems (representing human and machine computations) and allows the reduction of models of these axioms to Turing machines. Cellular automata and a variety of artificial neural nets can be shown to satisfy the axioms for machine computations.

Speaker: **Gabriel Uzquiano** (University of Oxford)

Title: *Ineffability and reflection*

Abstract: We know that not all concepts have extensions associated with them. In this talk, we would like to explore the hypothesis that a concept  $F$  lacks an extension if and only if  $F$  is ineffable, by which we mean, roughly, that no concept at least as large as  $F$  is describable by logical vocabulary alone. We are interested in this hypothesis largely because it seems to us to give partial expression to the inchoate thought that the universe is ineffable. Our first approximation to this thought will take the form of a second-order reflection schema on which, given a concept  $H$  at least as large as a concept  $F$  to which no extension corresponds, a sentence of pure second-order logic is true when relativized to the instances of  $H$  only if it is true when relativized to strictly fewer objects. We will look at the status of this schema vis-a-vis more familiar reflection principles.

One may be able to express the thought behind our reflection schema in finite compass by a sentence of a third-order language. However, once we allow ourselves the resources to do this, we find ourselves in a position to describe what is for a concept to be ineffable by the vocabulary of pure third-order logic, which betrays the very thought with which we started. This situation generalizes and we will look at the tension between, on the one hand, the drive to express the ineffability of the universe and, on the other, the constraint to remain faithful to it. (joint work with Stewart Shapiro)

Speaker: **Kai Wehmeier** (University of California, Irvine)

Title: *Identity is not a relation*

Abstract: Frege, Russell, and the early Wittgenstein all struggled with the notion of a binary relation that every object bears only to itself. In the *Tractatus* we even find an outright rejection of the notion, together with some gestures as to its eliminability from predicate logic. In the talk, I will sketch what seems to be the most promising argument against the existence of a binary relation of numerical identity, and discuss a few related logical issues.

Speaker: **Tim Williamson** (University of Oxford)

Title: *Adding probabilities to epistemic logic*



Abstract: The talk will use a case study to illustrate the philosophical interest of adding epistemic probabilities to standard possible world models of epistemic logic. It is familiar that the non-transitivity of the accessibility relation between worlds corresponds to the failure of the KK principle—if you know, you know that you know. How far can we turn the screw with counterexamples to the KK principle? That is, how low can your epistemic probability that you know  $p$  go at a world at which you do in fact know  $p$ ? Answer: As close to 0 as you like. Some of the relevant models can be instantiated in quite realistic settings. Implications will be considered for debates about the standard of epistemic warrant required for assertion and about apparent counterexamples to otherwise plausible closure principles for knowledge.

Speaker: **Byeong-Uk Yi** (University of Toronto)

Title: *Is logic axiomatizable?*

Abstract: I defend the negative answer to the question in the title, “Is logic axiomatizable?,” by considering sentences that involve plural constructions, such as the following:

- (A) There are some things each of which admires one of them.
- (B) There are some critics who admire only one another.

We can intuitively see that (A), for example, is logically implied by infinitely many sentences, such as the following:

- (A<sub>1</sub>)  $c_1$  admires  $c_2$ .
- (A<sub>2</sub>)  $c_2$  admires  $c_3$ .
- ⋮
- (A <sub>$n$</sub> )  $c_n$  admires  $c_{n+1}$ .
- ⋮

But (A) is not logically implied by any finite number of sentences among these. So the logic of languages that are rich enough to include (A) is non-compact. It follows from this that the logic of such languages is not axiomatizable. Similarly, we can see that (B), known as the Geach-Kaplan sentence, is logically implied by the following sentences (but not by any finite number of them):

- (A<sub>1</sub>)  $c_1$  admires only  $c_2$ ,  $c_1$  is not  $c_2$ , and  $c_1$  is a critic.
- (A<sub>2</sub>)  $c_2$  admires only  $c_3$ ,  $c_2$  is not  $c_3$ , and  $c_2$  is a critic.
- ⋮
- (A <sub>$n$</sub> )  $c_n$  admires only  $c_{n+1}$ ,  $c_n$  is not  $c_{n+1}$ , and  $c_n$  is a critic.
- ⋮

So we can conclude that the logic of languages that include (B) is not axiomatizable. To put the argument in proper perspective, I shall discuss contemporary account of plural constructions and suggest that they fail to do justice to the logic of plural constructions because they are based on the traditional view of plural constructions as devices for abbreviating singular constructions. And I shall give a sketch of my account of the logic of plural constructions that are based on the view of plurals as substantial devices that complement their singular cousins. In addition, I shall add brief remarks on how my argument for the non-axiomatizability of logic compares with the usual argument for the non-axiomatizability of second-order logic and with Tarski’s omega-consequence example in the beginning of his paper “On the concept of logical consequence,” and how it relates to David Kaplan’s proof of inexpressibility of (B) in elementary languages.